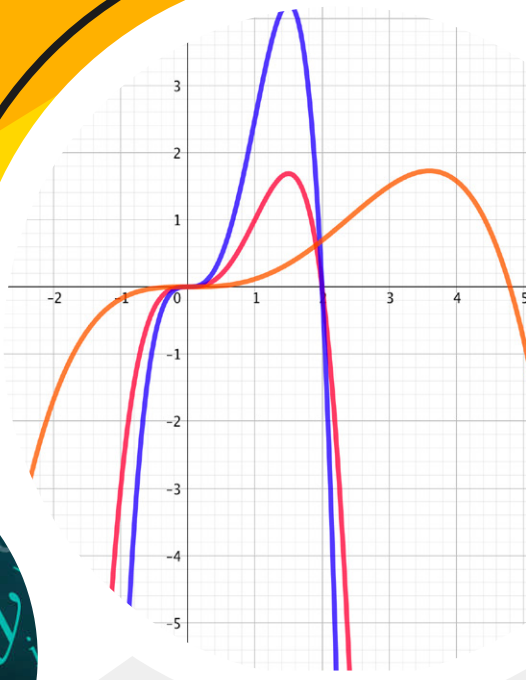
UNIVERSITÉ ABDERRAHMANE MIRA-BEJAIA
FACULTÉ DES SCIENCES EXACTES
LABORATOIRE DE MATHÉMATIQUES APPLIQUÉES

RDOPDE22

Recent Developments in Ordinary and Partial Differential Equations



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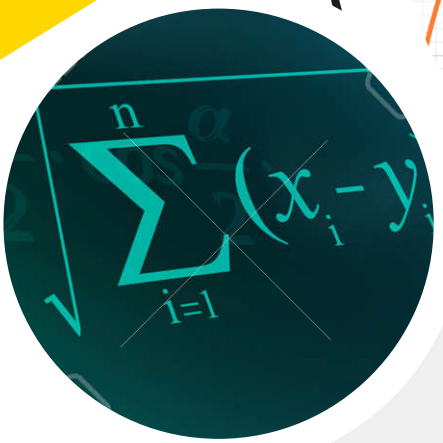

INVITED SPEAKERS

- Irena Lasiecka, Department of Mathematical Sciences, University of Memphis, USA.
- Mokhtar Kirane, University of La Rochelle, La Rochelle, France.
- Tian Xiang, Institute For Mathematical Sciences (IMS) at Renmin University of China.
- Hacène Belbachir, Université des Sciences et de la Technologie Houari Boumediene, Algeria.
- Nasser-eddine Tatar, King Fahd University of Petroleum and Minerals, Saudi Arabia.
- Miyasita Tosiya, Yamato University, Japan.
- Khaled Zennir, Department of Mathematics, College of Science and Arts, Al-Ras, Qassim University, Saudi Arabia.
- Mourad Sini, Radon Institute, Austrian Academy of Sciences, Austria.
- Svetlin Georgiev, University of Sofia, Bulgaria.
- Radu Precup, Babeş-Bolyai University, Cluj-Napoca, Romania.

CONFERENCE TOPICS

- Linear and nonlinear operators in function spaces.
- Differential, integral and operator equations.
- Initial and boundary value problems for ordinary and partial differential equations.
- Numerical methods for ordinary and partial differential equations.
- Mathematical and computer modeling.
- Mathematical physics and modeling in physics.

IMPORTANT DATES

- Registration deadline: 15 April 2022.
- Submission of Abstracts deadline: 15 April 2022.
- Notification for accepted abstracts: 03 May 2022.
- Conference program: 15 May 2022

SCIENTIFIC COMMITTEE

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- Loay Alkhalifa(Qassim University, Saudi Arabia)
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Recent Developments in Ordinary and Partial Differential Equations

RDOPDE 22 Bejaia, May 22-26 2022

Laboratory of Applied Mathematics
Bejaia University, Algeria

Laboratoire de Mathématiques Appliquées
UNIVERSITÉ de BEJAIA

Recent Developments in Ordinary and Partial Differential Equations

RDOPDE 22 Bejaia, May 22-26 2022

Bejaia University, Algeria

The main objective of this conference is to ensure participation of world-leading experts in diverse areas of Ordinary and Partial Differential Equations theory and applications. We also plan to attract a significant number of students and young researchers.

The academic program of the conference will consist of invited talks and oral communications. In the scheduled sessions, we will cover:

- Linear and nonlinear operators in function spaces.
- Differential, integral and operator equations.
- Initial and boundary value problems for ordinary and partial differential equations.
- Numerical methods for ordinary and partial differential equations.
- Mathematical and computer modeling.
- Mathematical physics and modeling in physics.

The organizers hope that RDOPDE will become a regular event and help create links between mathematicians. Due to the Covid 19 pandemic this edition will be held online.

Plenary Speakers

Pr. Irena Lasiecka, Department of Mathematical Sciences, University of Memphis, USA.

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Pr. Hacène Belbachir, Université des Sciences et de la Technologie Houari Boumediene, Algeria.

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Pr. Radu Precup, Babe?-Bolyai University, Cluj-Napoca, Romania.

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Rachid Boukoucha, Laboratory of Applied Mathematics, Bejaia University.

Sonia Medjbar, Laboratory of Applied Mathematics, Bejaia University.

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3. Dr. Tosiya Miyasita, Division of Mathematical Science, Department of Science and Engineering, Faculty of Science and Engineering, Yamato University, Osaka, Japan.
4. Pr. Mokhtar KIRANE, Department of Mathematics, College of Art and Sciences, Khalifa University of Science. and Technology, Abu Dhabi, United Arab Emirates.
5. Pr. Radu Precup, Babees-Bolyai University of Cluj-Napoca, Romania and Tiberiu Popoviciu Institute of Romanian Academy.
6. Pr. Mourad Sini, Radon Institute, Austria.
7. Pr. Svetlin G. Georgiev, Sofia University, Faculty of Mathematics and Informatics, Bulgaria.
8. Pr. Nasser-eddine Tatar, King Fahd University of Petroleum and Minerals, Department of Mathematics, Saudi Arabia.
9. Pr. Tian Xiang, Renmin University, China.
10. Pr. KHALED ZENNIR, Department of Mathematics, College of Sciences and Arts, Qassim University, Ar-Rass, Saudi Arabia.

Une extension de l'inégalité de Gram aux espaces L^{2n}

Hacène Balbachir

Université des Sciences et de la Technologie Houari Boumediene, Algeria

Résumé :
Il est bien connu que l'inégalité de Gram est une généralisation de l'inégalité de Schwarz en la considérant comme un déterminant. En introduisant le déterminant multidimensionnel sur des hyper matrices (tenseurs) symétriques, nous nous proposons de donner un équivalent de l'inégalité de Gram via une forme multilinéaire symétrique à 2n composantes. Ceci étant, cela nous permettra, durant l'exposé, de présenter une extension de l'identité de Lagrange.

RDOPDE 22
Recent Developments in Ordinary and Partial Differential Equations

Topic: Partial Differential Equations.

Long time behavior in flow-structure interactions

Irena Lasiecka

Department of Mathematical Sciences, University of Memphis.
E-mail: lasiecka@memphis.edu

Abstract: Flow-structure interactions are ubiquitous in nature. Problems such as attenuation of turbulence or flutter in an oscillating structure [Tacoma bridge], flutter in tall buildings, fluid flows in flexible pipes, in nuclear engineer- ing flows about fuel elements and heat exchanger vanes -are prime examples of relevant applications. Mathematically, the models are represented by a 3 D com- pressible, irrotational Euler Equation coupled to a nonlinear dynamic elasticity on a 2 D manifold. Strong boundary-type coupling at the interface between the two media is at the center of the analysis. This provides for a rich mathematical structure, opening the door to several unresolved problems in the area of non- linear PDE's, dynamical systems and related harmonic analysis and diifferential geometry. This talk aims at providing a brief overview of recent developments in the area along with a presentation of some recent advances addressing the issues of control and long time behavior [partial structural attractors] subject to mixed boundary conditions arising in modeling of the interface between the two environments.
Keywords: Euler Equation, nonlinear elasticity, flutter, stabilitzation, compact attractors.

Remarks on radial solutions of a parabolic Gelfand-type equation

Tosiya MIYASITA

Abstract

We consider an equation with exponential nonlinearity. In this talk, we concentrate on the radial solutions. First, we find a global solution for sufficiently small initial value and parameter by Sobolev embedding and Poincaré inequalities together with a decreasing energy. Next, we study the corresponding elliptic equation and treat its spectral property.

1 Introduction

In [1, 2], we consider a parabolic equation

$$\begin{cases} u_t = \Delta u + \lambda\,(e^u - 1) & x \in \Omega,\ \ t \in (0, T_{u_0}), \\ u(x,t) = 0 & x \in \partial\Omega,\ \ t \in (0, T_{u_0}), \\ u(x,0) = u_0(x) & x \in \Omega, \end{cases} \tag{1}$$

where $\lambda > 0$, Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$ for $n \in \mathbb{N}$ and T_{u_0} denotes the maximal existing time of the local solution for an initial function u_0. In [2], the author established a unique global solution for sufficiently small $\lambda > 0$ and $u_0 \in H_0^1(\Omega)$ with $n = 1, 2$. To prove the results, first of all, we derive the energy inequality from Lyapunov function. Next we apply Sobolev embedding theorem for $n = 1$ and Trudinger-Moser inequality for $n = 2$. Thus it is not easy to extend the results in [2] for $n \geq 3$. In this talk, we assume that the domain is an annulus $A_a \equiv \{x \in \mathbb{R}^n \mid a < |x| < a^{-1}$ for $0 < a < 1$ and $n \geq 3\}$ and concentrate on the radial solutions $u(r) = u(|x|)$ for $r = |x|$. Then problem (1) is reduced

Mathematics subject classification(2020): 35K58, 35J61, 35B45, 35B32, Keywords and Phrases: Exponential nonlinearity, Lyapunov function, Global solution, Stationary solution.

to

$$\begin{cases} u_t = u_{rr} + \frac{n-1}{r} u_r + \lambda\,(e^u - 1) & r \in (a, a^{-1}),\ \ t \in (0, T_{u_0}), \\ u(a,t) = u(a^{-1}, t) = 0 & t \in (0, T_{u_0}), \\ u(r,0) = u_0(r) & r \in (a, a^{-1}). \end{cases} \tag{2}$$

Note that any interval $\alpha < s < \beta$ in $\mathbb{R}$ is transformed into $a < r < a^{-1}$ through the relation $r = (\alpha\beta)^{-1/2} s$ and $a = \alpha^{1/2}\beta^{-1/2}$. Hence the problem on any interval is equivalent to that on (a, a^{-1}). Henceforward, we denote $I \equiv (a, a^{-1})$ and $|I| = a^{-1} - a$, respectively. We denote the H_0^1 space with respect to r by $\mathcal{H} = H_0^1(a, a^{-1})$.

Nowadays, it seems that there are not enough studies for (1). If Ω is a unit ball, in [1] they study the bifurcation diagram of the stationary positive solution and compute the bound for Morse index globally, not locally around a bifurcation point. If the solution is positive and radially symmetric, they establish the existence of singular solution, multiple existence of the regular solution and bound for its Morse index. In [2], he deals on the bifurcation diagram of the stationary solution, not always positive, for $n = 1$, proves that nontrivial solutions bifurcate from trivial solution and computes the Morse index locally around each bifurcation point. He finds blow-up criteria and proves the existence of a global solution for sufficiently small initial value and parameter. The aim of this talk is to make a few remarks for the solution of (1) for higher dimensional case. Similarly to the one dimensional case, by Lyapunov function and Sobolev embedding theorem, we can prove the following theorem on the existence of a unique global solution for sufficiently small $\lambda > 0$ and $u_0 \in \mathcal{H}$.

Theorem 1 *Let $n \geq 3$ and $0 < a < 1$. If $u_0 \in \mathcal{H}$ and λ are sufficiently small, then there exists a unique global solution of (2) satisfying*

$$u \in C\big((0,+\infty); \mathcal{H} \cap H^2(I)\big) \cap C\big([0,+\infty); \mathcal{H}\big) \cap C^1\big((0,+\infty); L^2(I)\big).$$

Next, we consider the stationary problem corresponding to (1), that is,

$$\begin{cases} \Delta v + \lambda\,(e^v - 1) = 0 & x \in \Omega, \\ v(x) = 0 & x \in \partial\Omega. \end{cases}$$

Similarly, we treat the radial problem

$$\begin{cases} v_{rr} + \frac{n-1}{r} v_r + \lambda\,(e^v - 1) = 0 & r \in I, \\ v(a) = v(a^{-1}) = 0. \end{cases} \tag{3}$$

We define the solution set $\mathcal{C}$ by

$$\mathcal{C} \equiv \Big\{(\lambda, v) \in \mathbb{R}^+ \times \big(C^2(I) \cap C_0(\overline{I}\big) \;\Big|\; v = v(r) \text{ solves (3) for } \lambda > 0\Big\},$$

where $\mathbb{R}^+ = \{k \mid k > 0\}$ and

$$C_0(\overline{I}) \equiv \Big\{v \in C(\overline{I}) \;\Big|\; v(a) = v(a^{-1}) = 0 \;\Big\}$$

endowed with the L^∞ norm. Similarly to [2], we have the following bifurcation result:

Theorem 2 *Let $n \geq 3$ and $0 < a < 1$. There exists a sequence κ_i with*

$$0 < \kappa_1 < \kappa_2 < \cdots < \kappa_i < \cdots <\uparrow +\infty$$

such that two continua $\mathcal{S}_i^{\pm} \subset \mathcal{C}$ of nontrivial solution of (3) *bifurcate at* $(\lambda, v) = (\kappa_i, 0)$.

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Tosiya Miyasita
Division of Mathematical Science, Department of Science and Engineering, Faculty of Science and Engineering, Yamato University, 2-5-1, Katayama-cho, Suita-shi, Osaka, 564-0082, Japan
e-mail: miyasita.t@yamato-u.ac.jp

RDOPDE 22
Recent Developments in Ordinary and Partial Differential Equations

Topic: Fractional Calculus

A SURVEY OF USEFUL INEQUALITIES IN FRACTIONAL CALCULUS

Mokhtar KIRANE

Department of Mathematics, College of Art and Sciences, Khalifa University of Science and Technology, Abu Dhabi, United Arab Emirates,
E-mail: mokhtar.kirane@yahoo.com

Abstract: A survey on inequalities in fractional calculus that have proven to be very useful in analyzing differential equations is presented. We mention in particular, a Leibniz inequality for fractional derivatives of Riesz, Riemann-Liouville or Caputo type and its generalization to the d-dimensional case that become a key tool in differential equations; they have been used to obtain upper bounds on solutions leading to global solvability, to obtain Lyapunov stability results, and to obtain blowing-up solutions via diverging in a finite time lower bounds. We will also mention the weakly singular Gronwall inequality of Henry and its variants, principally by Medved, that plays an important role in differential equations of any kind. We will also recall some traditional inequalities involving fractional derivatives or fractional powers of the Laplacian.
Keywords: fractional calculus, inequalities

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RDOPDE 22
Recent Developments in Ordinary and Partial Differential Equations

Topic: Ordinary Differential Equations, Nonlinear Operator Theory

Nonlinear alternatives of hybrid type for nonself vector-valued maps and application to differential systems

Radu Precup

Babeş-Bolyai University of Cluj-Napoca, Romania
and
Tiberiu Popoviciu Institute of Romanian Academy
E-mail: `r.precup@math.ubbcluj.ro`

Abstract: The lecture is devoted to nonlinear alternatives of Leray-Schauder and Mönch type for nonself vector-valued operators, under hybrid conditions of Perov contraction and compactness. Thus we present vector versions of the theorems of Krasnosel'skii, Avramescu, O'Regan, Burton-Kirk and Gao-Li-Zhang. An application is given to a boundary value problem for a system of second order implicit differential equations.

Keywords: nonlinear operator; nonself map; fixed point; Perov contraction; nonlinear boundary value problem.

1 Introduction

Any study in operator equations with hybrid conditions must begin with Krasnosel'skii's theorem for the sum of two operators.

Theorem 1 (Krasnosel'skii)**.** [7] *Let D be a closed bounded convex subset of a Banach space X, $A : D \to X$ a contraction and $B : D \to X$ a continuous mapping with $B(D)$ relatively compact. If*

$$A(x) + B(y) \in D \quad \text{for every} \quad x, y \in D, \tag{1}$$

then the map $N := A + B$ has at least one fixed point.

The hybrid character of Krasnosel'skii's theorem lies in the decomposition of the operator N as a sum of two maps A and B with different properties. An other possibility for a hybrid approach arises in case of systems, when the domain of N splits as a Cartesian product, say $X \times Y$, and correspondingly the operator N splits as a couple (N_1, N_2), where N_1, N_2 take their values in X and Y, respectively. A typical result in this direction is the following vector version of Krasnosel'skii's theorem, due to Avramescu.

Theorem 2 (Avramescu). [1] *Let $(D_1,\ d)$ be a complete metric space, D_2 a closed convex subset of a normed space Y and let $N_i : D_1 \times D_2 \to D_i$, $i = 1, 2$ be continuous mappings. Assume that the following conditions are satisfied:*

(a) *There is a constant $l \in [0, 1)$ such that*

$$d\left(N_1\left(x, y\right),\ N_1\left(\overline{x}, y\right)\right) \leq l d\left(x,\ \overline{x}\right)$$

for all $x, \overline{x} \in D_1$ and $y \in D_2$;

(b) *$N_2\left(D_1 \times D_2\right)$ is a relatively compact subset of Y.*

Then there exists $(x, y) \in D_1 \times D_2$ with $N_1\left(x, y\right) = x, \quad N_2\left(x, y\right) = y$.

Extensions of Krasnosel'skii's theorem to nonself maps have been given by O'Regan [8] (1996), Burton-Kirk [3] (1998) and Gao-Li-Zhang [5] (2011).

The presentation is mainly based on paper [6]. Related results can be found in [2] and [4].

2 Abstract results for nonself maps

First we obtain an Avramescu type principle for nonself maps. Consider a system of two operator equations

$$\begin{cases} N_1\left(x, y\right) = x \\ N_2\left(x, y\right) = y. \end{cases} \tag{2}$$

Theorem 3. *Let Y be a Banach space, $K \subset Y$ a retract of Y and $U \subset K$ open in K. Let Λ be a topological space and*

$$N_1 : \Lambda \times \overline{U} \to \Lambda, \qquad N_2 : \Lambda \times \overline{U} \to K$$

be two mappings such that the following conditions are satisfied:

(a) *For each $y \in \overline{U}$, there is a unique $x =: S\left(y\right) \in \Lambda$ with*

$$N_1\left(S\left(y\right), y\right) = S\left(y\right);$$

(b) *There is a compact map $H : \overline{U} \times [0, 1] \to K$, $H_\lambda := H\left(\cdot, \lambda\right)$, with*

$$i\left(H_0,\ U,\ K\right) \neq 0 \quad \text{and} \quad H_1 = N_2\left(S\left(.\right), .\right).$$

Then either

(i) *the system (2) has a solution $(x, y) \in \Lambda \times \overline{U}$, or*

(ii) *there is a point $y \in \partial_K U$ and $\lambda \in (0, 1)$ with $y = H\left(y, \lambda\right)$.*

Theorem 3 gives in particular hybrid results for nonself maps of Krasnosel'kii, O'Regan, Burton-Kirk and Gao-Li-Zhang types. Instead the common contraction property, we consider its vector analogue, the Perov contraction.

3 Application

An application of the vector version of Burton-Kirk theorem, is given to the following boundary value problem for a system of n equations

$$\begin{aligned} -u_i'' &= f_i\left(t, V_i u\right) + g_i\left(t, V_0 u\right), \quad \text{a.e. } t \in (0,1) \\ u_i\left(0\right) &= u_i\left(1\right) = 0, \quad i = 1, 2, \cdots, n, \end{aligned} \tag{3}$$

where $V_0 u$ and $V_i u$ denote the vectors

$$V_0 u = (u, u') = (u_1, \cdots, u_n, u_1', \cdots, u_n'), \qquad V_i u = (V_0 u,\ u_i'') = (u, u', u_i'').$$

Note that the equations are implicit due to the dependence on u_i'' of the terms $f_i\left(t, V_i u\right)$.

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Topic: Partial differential Equations and Applications

Mathematics Of The Imaging Modalities Using Contrast Agents

Mourad Sini

Radon Institute, Altenbergerstrasse 69, A-4040, Linz, Austria
E-mail: mourad.sini@oeaw.ac.at

Abstract: It is, nowadays, a common certainty that the inverse problems of recovering objects from remote measurements are, mostly, highly unstable, especially for low contrasting tissues (as tumors in early stages) or fluids. To recover the stability, it is advised in the engineering literature to create, whenever possible, the missing contrasts in the targets to image by injecting contrast-agents. In this talk, we follow this direction and propose an approach how to analyze mathematically the effect of the injected agents on the different fields under consideration. These contrast agents are small-sized particles modeled with materials that enjoy high contrasts as compared to the ones of the background. These two properties allow them, under critical scales of size/contrast, to create local spots when excited from far. These local spots can be remotely recovered in stable ways. The accessible information on the target to image are encoded in theses spots.

We consider two modalities, namely the acoustic imaging and photo-acoustic imaging, where the contrast agents are micro-bubbles and nano-particles respectively. In these cases, we provide a clear and useful correspondence between the critical size/contrast scales and the main resonances, and hence the local spots, they are able to create while excited with appropriate incident frequencies.

1. Acoustic imaging using bubbles. From the remotely measured back-scattered fields, we recover the (Minnaert) resonance and the total field on the location of the bubble. From the Minnaert resonance, we reconstruct the mass density and from the total field we reconstruct the bulk modulus of the target to image.

2. Photo-acoustic using nano-particles. Here we have two, connected, waves: the time domain acoustic wave and electromagnetic wave. From the measured acoustic wave, we first recover the internal travel-time function and then the electric field (on the nano-particle). From the travel time, we reconstruct the acoustic speed of propagation, via the Eikonal equation, and from the electric field, we recover the plasmonic resonant frequencies from which we reconstruct the permittivity.

Keywords: Photo-acoustics, Electromagnetism, micro-bubbles, nano-particles, plasmonic resonances, dielectric resonances.

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RDOPDE 22
Recent Developments in Ordinary and Partial Differential Equations

Topic: Ordinary Differential Equations

Boundary Value Problems for First Order Impulsive Dynamic Equations

Svetlin G. Georgiev

Sofia University, Faculty of Mathematics and Informatics, Department of Differential Equations, Bulgaria
E-mail: svetlingeorgiev1@gmail.com

Abstract: This lecture is devoted to a qualitative analysis of some classes of boundary value problems for first order impulsive dynamic equations. Lower and upper solutions for first order impulsive dynamic equations are studied. They are investigated for existence and nonuniqueness of solutions of first order impulsive dynamic equations with general boundary conditions and periodic boundary conditions. Criteria for existence of extremal solutions are given.
Keywords: first order impulsive dynamic equations, existence of solutions, uniqueness of the solutions, boundary value problems, periodic boundary value problems.

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Topic: Fractional Differential Equations

From integer-order PDEs to fractional-order PDEs

Nasser-eddine Tatar

King Fahd University of Petroleum and Minerals,
Department of Mathematics,
Dhahran 31261, Saudi Arabia
E-mail: tatarn@kfupm.edu.sa

Abstract: In this talk, we will justify and explain the passage from integer-order PDEs to fractional-order PDEs. The subordination principle which allows to inherit the well-posedness of the fractional problem from the corresponding integer one will be presented. Moreover, we will stability issue as well as some open problems.

Keywords: Fractional derivative; well-posedness; stability

Introduction:

In many complex diusion phenomena, the mean square displacement does not grow linearly as in the ordinary cases. It grows nonlinearly in time as a power function in case of disordered systems. This suggests using fractional dierential models to describe the evolution of such processes. This represents an appropriate alternative to the costly nonlinear models. In this talk, we will go over some problems in viscoelasticity, thermoelasticity and porous media. The well-posedness as well as the stability of such systems with the involved integer-order derivatives replaced by fractional ones will be discussed. In particular, we will explain how we can pass smoothly from the integer case to the non-integer case. The main difficulties and some open problems will be highlighted.

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On Krasnosels'kii type fixed point theorems

Tian Xiang, txiang@ruc.edu.cn

Renmin University of China

Abstract: A nice combination of the well-known Banach fixed point theorem for self-contraction map and the Schauder fixed point theorem for compact and continuous map yields the classical Krasnosel'skii fixed point theorem. In this talk, we survey various developments of the Krasnosel'skii type fixed point theorem and indicate their applications in integral, differential and difference equations etc.

Mots-Clés: Krasnosel'skii type fixed point theorems, contraction, expansion, measure of noncom-pactness, measure of weak noncompactness.

STABILIZATION FOR SOLUTIONS OF PLATE EQUATION WITH TIME-VARYING DELAY AND WEAK-VISCOELASTICITY IN $\mathbb{R}^n$

KHALED ZENNIR

Abstract. This talk considers a dynamical system with delay described by a differential equation with partial derivatives of hyperbolic type and delay with respect to a time variable. We established the $k(t)$-stability of weak solution under suitable initial conditions in $\mathbb{R}^n, n > 4$ by introducing an appropriate Lyapunov functions.

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Khaled zennir
Department of Mathematics, College of Sciences and Arts, Qassim University, Ar-Rass, Saudi Arabia
E-mail address: khaledzennir4@gmail.com

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Key words and phrases. Plate equation; Weak-viscoelastic; Variable delay; Energy decay; Weighted space; Density.

Ordinary Differential Equations and Continuous Dynamical Systems

A class of differential problems in Banach spaces

Fatiha Selamnia [(1)], Dalila Azzam-Laouir [(2)]

[(1)] Université de Jijel, Algérie

E-mail: `fatihamenigher@gmail.com`

[(2)] Université de Jijel, Algérie

E-mail: `laouir.dalila@gmail.com`

Abstract: In this talk we prove, in separable Banach space E, the existence of solutions for the sweeping process of the form

$$-\dot{u(t)} \in N_{C(t,u(t))}(u(t)) + F(t, u(t)), \ , a.e. \ , t \in [0, T],$$

where $F : [0, T] \times E \to E$ is an upper semi-continuous set-valued mapping with nonempty closed convex values.

$C : [0, T] \times E \to E$ is a set-valued mapping with nonempty, ball compact and r-prox-regular and $N_{C(t,x)}(.)$ is the proximal normal cone of C(t,x).

The existence of solutions for sweeping processes has been stud- ied by many authors since the pioneering work by J.J. Moreau in the 70's (see [2]). He expressed that sweeping process by the following evolution differential inclusion

$$-\dot{u}(t) \in N_{C(t)}(u(t)), \text{ a.e. } t \in [0, T]; \ u(0) = u_0 \in C(0),$$

where $C(t)$ is a closed convex set in a Hilbert space H and $N_{C(t)(.)}$ is the normal cone to $C(t)$ in the sense of convex analysis, see also [3]. Then, some contributions in the context of nonconvex sets $C(t)$ were given in a series of papers, see for instance [1], [4].

Let $I = [0, T]$ $(T >)$ and E be a separable, reflexive, uniformly smooth Banach space which is $I-$smoothly weakly compact for an exponent $p \in [2, \infty]$. Let $F : I \times E \to E$ be a set-valued mapping with nonempty convex weakly compact values such that

(H_F^1) F is scalarly $\mathcal{L}(I) \otimes \mathcal{B}(E)$-measurable, that is for each $e \in E$, the scalar function $\delta^*(e, F(., .))$ is $\mathcal{L}(I) \otimes \mathcal{B}(E)$-measurable;

(H_F^2) for each $t \in I$, $F(t,.)$ is scalarly upper semicontinuous, that is for each $e \in E$, the scalar function $\delta^*(e, F(t,.))$ is upper semicontinuous on E;

(H_F^3) il existe une real constant m positive tel que

$$|P_{F(t,x)}(0)| = d(0, F(t,x)) \leq m, \ ; \forall (t,x) \in I \times E.$$

Let $r > 0$ and $C : I \times E \to E$ be a set-valued mapping taking nonempty closed and $r-$prox-regular values. We assume that the following assumptions are satisfied.

(H_C^1) There are real constants $k_1 > 0, 0 \leq k_2 < 1$ such that for all $s, t \in I$ and $u, v, x \in E$

$$|d(x, C(t,u)) - d(x, C(s,v))| \leq k_1|t-s| + k_2\|u-v\|;$$

(H_C^2) For any bounded $A \subset E$, the set $C(I \times A)$ is relatively ball compact, i.e., the intersection of $C(I \times A)$ with any closed ball is relatively compact.

Then for any $u_0 \in (0, u_0)$, the differential inclusion

$$(\mathcal{P}_F) \begin{cases} u(0) = u_0; \\ u(t) \in C(t, u(t)), \quad \forall t \in I; \\ -\dot{u}(t)) \in N_{C(t,u(t))}(u(t)) + F(t, u(t)), \quad a.e.\ t \in [0,T]; \end{cases}$$

has a Lipschitz solution $u : I \longrightarrow E$.

Keywords: Ball-compactness, differential inclusions, r-prox-regularity, set-valued mapping, sweeping process, upper semi-continuity.

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Approximate–null controllability for semilinear differential inclusion in abstract Banach spaces

BENNICHE Omar [(1)]

[(1)] Departement of mathematics, Djilali BOUNAAMA University

E-mail: `o.benniche@univ-dbkm.dz`

Abstract: We provide sufficient conditions for approximate null–controllability for non–parametric semilinear differential inclusions of the form $y' \in Ay + F(t, y(t))$ where $A : D(A) \subset X \to X$ is a linear operator generating a C_0–semigroup and $F : [a, b] \times X$

$\rightsquigarrow X$ is a given set–valued map where $-\infty < a < b \leq +\infty$.

Given a real Banach space $(X, \|\cdot\|)$, consider the following non–parametric control system:

$$y'(t) \in Ay + F(t, y), \tag{1}$$

where $A : D(A) \subset X \to X$ is the infinitesimal generator of a C_0–semigroup $\{S(t) : X \to X; t \geq 0\}$ and $F : [a, b] \times X \rightsquigarrow X$ is a given set–valued map where $-\infty < a < b \leq +\infty$.

By a solution of (1) on $[t_0, T) \subset [a, b)$ we mean a continuous function $y : [t_0, T] \to X$ for which there exists a pseudoderivative $f_y \in L^1(t_0, T; X)$ satisfying $f_y(t) \in F(t, y(t))$ a.e. on $[t_0, T]$ and such that

$$\forall t \in [t_0, T), y(t) = S(t - t_0)y(t_0) + \int_{t_0}^{t} S(t - s)f_y(s)ds.$$

Here, we investigate approximate null–controllability of the couple: The Banach space X and the system (1). Roughly speaking, we say that $x \in X$ is approximate null–controllable on $[s_0, T] \subset [a, b)$ with respect to (1) if for every $\epsilon > 0$, there exists a solution $y_\epsilon : [s_0, T] \to X$ of (1) with $y_\epsilon(s_0) = x$ and $\|y_\epsilon(T)\| \leq \epsilon$.

Our approach is based on the global near weak invariance of a set S with respect to a dynamical system. We recall that near weak invariance means the existence of solutions which remain arbitrarily close to a given set starting from initial states in that set whereas (exact) weak invariance means the existence of solutions which remain in a given set starting from that set.

Near weak invariance has been characterized in terms of tangency conditions. Unlike the (exact) weak invariance, convexity or compactness of the values of F is not needed. Furthermore, the compactness of the semigroup may be dropped.

Our main result consists to prove that approximate null–controllability of (1) can be obtained from global near viability of the epigraph of the norm of X with respect to a related dynamical system. Then, we show that the non–parametric control system (1) is approximately–controllable under a new weak Petrov condition which assumes a semi scalar product to be bounded away from zero by a term that is not required to be constant as in previous works.

As application, we give an approximate null– controllability result for a class of abstract diffusion-reacted system.

Keywords: Approximate null–controllability, Weak invariance, Differential inclusion.

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Existence of Solutions for a Class of Boundary Value Problems for Weighted p(t)-Laplacian Impulsive Systems

Kamilia Khemmar [(1)], Karima Mebarki [(1)], Svetlin G. Georgiev[(2)]

[(1)] Laboratory of Applied Mathematics, Faculty of Exact Sciences, Bejaia University, 06000 Bejaia, Algeria.

E-mail: `kamiliamaths@gmail.com`

E-mail: `mebarqi_karima@hotmail.fr`

[(2)] Department of Dierential Equations, Faculty of Mathematkics and Informatics, University of Sofia, Sofia, Bulgaria.

E-mail: `svetlingeorgiev1@gmail.com`

Abstract: In this talk, we investigate the weighted $p(t)$-Laplacian system

$$-\left(w(t)|x'(t)|^{p(t)-2}x'(t)\right)' + f\left(t, x(t), (w(t))^{\frac{1}{p(t)-1}}x'(t)\right) = 0, \quad t \in (0,T),\ t \neq t_j, \tag{2}$$

where $x : [0,T] \to \mathbb{R}^N$, $N \geq 1$, with the following impulsive boundary conditions

$$x(t_j^+) - x(t_j) = A_j\left(t_j, x(t_j), (w(t_j))^{\frac{1}{p(t_j)-1}}x'(t_j)\right), \quad j \in \{1,\dots,k\}, \tag{3}$$

$$\begin{aligned} w(t_j^+)|x'(t_j^+)|^{p(t_j^+)-2}x'(t_j^+) &= w(t_j)|x'(t_j)|^{p(t_j)-2}x'(t_j) \\ &\quad + B_j\left(t_j, x(t_j), (w(t_j))^{\frac{1}{p(t_j)-1}}x'(t_j)\right),\ j \in \{1,\dots,k\}, \end{aligned} \tag{4}$$

$$ax(0) - b(w(0))^{\frac{1}{p(0)-1}}x'(0) = 0, \tag{5}$$

$$cx(T) + dw(T)|x'(T)|^{p(T)-2}x'(T) = 0, \tag{6}$$

where

(H1) $f \in \mathcal{C}([0,T] \times \mathbb{R}^N \times \mathbb{R}^N)$,

$$|f(t,x,y)| \leq a_1(t) + a_2(t)|x|^{p_1} + a_3(t)|y|^{p_2}, \quad t \in [0,T], \quad x, y \in \mathbb{R}^N,$$

$a_1, a_2, a_3 \in \mathcal{C}([0,T])$, $0 \leq a_1, a_2, a_3 \leq B$ on $[0,T]$ for some constant $B > 1$, $p_1, p_2 \geq 0$.

(H2) $p \in \mathcal{C}([0,T])$, $w \in \mathcal{C}^1([0,T])$, $p > 1$, $w > 0$ on $[0,T]$,

$$w(t) \leq B, \quad p(t) \leq B, \quad (w(t))^{\frac{1}{p(t)-1}} \leq B, \quad t \in [0,T],$$

$a, b, c, d \in \mathbb{R}$, $0 = t_0 < t_1 < \ldots, t_k < t_{k+1} = T$, $k \in \mathbb{N}$.

(H3) $A_j \in \mathcal{C}([0,T] \times \mathbb{R}^N \times \mathbb{R}^N)$,

$$|A_j(t,x,y)| \leq a_{1j}(t) + a_{2j}(t)|x|^{p_{1j}} + a_{3j}(t)|y|^{p_{2j}}, \quad j \in \{1,\ldots,k\},$$

$0 \leq a_{1j}, a_{2j}, a_{3j} \leq B$ on $[0,T]$, $p_{1j}, p_{2j} \geq 0$, $j \in \{1,\ldots,k\}$.

(H4) $B_j \in \mathcal{C}([0,T] \times \mathbb{R}^N \times \mathbb{R}^N)$,

$$|B_j(t,x,y)| \leq b_{1j}(t) + b_{2j}(t)|x|^{q_{1j}} + b_{3j}(t)|y|^{q_{2j}}, \quad j \in \{1,\ldots,k\},$$

$0 \leq b_{1j}, b_{2j}, b_{3j} \leq B$ on $[0,T]$, $q_{1j} \geq 0$, $q_{2j} \geq 0$, $j \in \{1,\ldots,k\}$.

Define

$$\begin{aligned} PC^2([0,T]) \;=\; & \big\{ g : [0,T] \to \mathbb{R}^N, \quad g \in \mathcal{C}^2([0,T] \backslash \{t_j\}_{j=1}^k) \,, \\ & g^{(i)}(t_j^-),\ g^{(i)}(t_j^+) \ \text{ exist} \quad \text{and} \quad g^{(i)}(t_j^-) = g^{(i)}(t_j), \quad j \in \{1,\ldots,k\}, \quad i \in \{0,1,2\} \big\} \,. \end{aligned}$$

Below suppose that $A > 0$ and $C > 0$ are constants so that: **(H5)** $C(1 + T + T^2) \leq A$. For $\epsilon \in (0,1)$, we suppose that the constants B and A which appear in the conditions $(H1)$ and $(H5)$, respectively, satisfy the following inequality: **(H6)** $\epsilon B_1(1 + A) < B$ and $AB_1 < B$.

Our main result claims that if $(H1)$-$(H6)$ hold, then the problem (1)-(3) has at least one solution in $PC^2([0,T])$. To prove this result we propose a new approach based upon recent theoretical results on the fixed point theorem for the sum of two operators.

Existence result for a non-autonomous second order difference equations via a new fixed point theorem

Lydia Bouchal [(1)], Karima Mebarki [(2)], Svetlin Georgiev Georgiev [(3)]

[(1)] *Laboratory of Applied Mathematics, Faculty of Exact Sciences, University of Bejaia, 06000 Bejaia, Algeria.*
E-mail: `lydia.bouchal@univ-bejaia.dz`

[(2)] *Laboratory of Applied Mathematics, Faculty of Exact Sciences, University of Bejaia, 06000 Bejaia, Algeria.*
E-mail: `karima.mebarki@univ-bejaia.dz`

[(3)] *Department of Differential Equations, Faculty of Mathematics and Informatics, University of Sofia, Sofia, Bulgaria.*
E-mail: `svetlingeorgiev1@gmail.com`

Abstract: In this talk we establish a new result on the existence of at least one positive solution to the following non-autonomous second order difference equation:

$$\triangle^2 u(k) + f(k, u(k)) = 0, \quad k \in \{0, 1, \ldots, N\},\ N \in \mathbb{N}, N > 1, \tag{7}$$

with boundary conditions

$$u(0) = u(N+2) = 0, \tag{8}$$

where $\triangle^2$ is the second forward difference operator which acts on u by $\triangle^2 u(k) = u(k+2) - 2u(k+1) + u(k),\ k \in \{0, 1, \ldots, N\}$ and $f : \{0, ..., N+2\} \times [0, \infty) \to [0, \infty)$ is a continuous function satisfying:

$$(\mathcal{H}_1)\text{:} \begin{cases} 0 \le f(k, u(k)) \le a(k) + b(k)|u(k)|^p,\ p \ge 0,\ a, b : \{0, \ldots, N+2\} \to [0, \infty) \text{ be such that} \\ 0 \le a(k),\ b(k) \le B,\ k \in \{0, \ldots, N+2\}, \text{ for some positive constant } B. \end{cases}$$

By positive solution, we mean a function $u : \{0, \ldots, N+2\} \to \mathbb{R}$ such that $u(k) \ge 0$ on $\{0, 1, \ldots, N+2\}$ and satisfies the posed BVP.

Suppose

$(\mathcal{H}_2)$**:** ϵ, A_1, B, B_1, R, R_1, r are positive constants such that

$$\epsilon \in (0,1),\ \frac{B_1}{2} > A_1(N+3)\left((N+2)(N+1)B\left(1+R^p\right)+R\right),$$

$$\frac{r}{A_1} < R, \quad R_1 > \max\{R,1\}, \quad A_1 \in (0,1),$$

$$A_1(\epsilon + r + 2B_1) \leq r.$$

The main result is as follows.

Theorem: Suppose that $(\mathcal{H}_1)$ and $(\mathcal{H}_2)$ hold. Then the BVP (1)-(3) has at least one positive solution $u^* \in E$ such that $A_1 \max\limits_{k\in\{0,\dots,N+2\}} u^*(k) \geq r$ and $\max\limits_{k\in\{0,\dots,N+2\}} u^*(k) \leq R$.

The approach used is the fixed point theory in cones of a Banach space. Precisely, we develop a new fixed point theorem of functional type for the sum of two operators $T+S$ where $I-T$ is Lipschitz invertible and S a k-set contraction. The arguments are based upon of recent results on the fixed point index developed in [4] and [5].

Keywords: Fixed point; sum of operators; non-autonomous difference equations; positive solution.

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Existence Results To Time Optimal Control Problems With Almost Convex Condition

Imen Boutana[(1)], Azzam-Laouir Dalila[(2)]

[(1)] LAOTI Laboratory, Jijel University, Algeria
E-mail: i_boutana@univ-jijel.dz

[(2)] LAOTI Laboratory, JijelUniversity, Algeria
E-mail: laouir.dalila@gmail.com

Abstract: This talk is devoted mainly to prove the existence of absolutely continuous solutions to time differential inclusion under almost-convex condition, which is a strictly weaker condition than the usual assumption of convexity on the values of the right-hand side.

Given $J = [0, T] \subset \mathbb{R}$ and $\Omega \subset \mathbb{R}^n$ be closed. For a subset $K \subset \mathbb{R}^n$, $co(K)$ denote the convex hull of K.

Let X be a vector space. A set $K \subset X$ is calles almost convex if for every $\xi \in co(k)$ there exist λ_1 and λ_2, $0 \leq \lambda_1 \leq 1 \leq \lambda_2$, such that $\lambda_1\xi \in K,\ \lambda_2\xi \in K$.

The existence of solutions for the first order differential inclusions of the form

$$(\mathcal{P}_F) \begin{cases} -\dot{u}(t) \in Au(t) + F(t, u(t)) \quad a.e. t \in J, \\ u(0) = u_0 \end{cases}$$

which is the subject of our interest, vary a lot according to the hypotheses imposed on the multi-application F. The convexity hypothesis is widely used, in the calculus of variations in optimal control, and in differential inclusions to prove the existence of solutions, particularly to establish the closure of the set of solutions, which is generally unclosed without convexity. The problem $(\mathcal{P}_F)$ has been studied by Dalila Azzam-Laouir and all [2] in the case where F is an upper semicontinuous multifunction with convex values. The non-convex case has been studied by various approaches. Note that in [4], a generalization of convexity has been defined, namely, the almost convexity of sets. This almost convexity condition has been used successfully by many authors, see for example [1]. Our aim in this talk, is to provide in finite dimensional space the existence of absolutely continuous solutions for the problem $(\mathcal{P}_F)$ where $F : \Omega \times \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ $(\Omega \subset \mathbb{R}^n)$ is a multifunction, upper

semicontinuous with almost convex closed values.

Theorem 1.

Let $F : J \times \Omega \rightrightarrows \mathbb{R}^n$ be an almost convex closed valued multifunction, satisfying the following assumptions:

1. For For each t, $F(t,.)$ is scalarly $(\mathcal{L}(J) \otimes \mathcal{B}(\mathbb{R}^n))$-measurable, i.e., for each $t \in J$ and each $e \in \mathbb{R}^n$, the scalar function $\delta^*(e, F(t,.))$ is $(\mathcal{L}(J) \otimes \mathcal{B}(\mathbb{R}^n))$-measurable;

2. for each t, $F(t,.)$ is scalarly upper semicontinuous on $\mathbb{R}^n$, i.e., for each $t \in J$ and each $e \in \mathbb{R}^n$, the scalar function $\delta^*(e, F(t,.))$ is upper semicontinuous on $\mathbb{R}^n$;

3. $Proj_{F(t,x)}(0) \subset (1 + \|x\|)C, \quad \forall (t,x) \in J \times \mathbb{R}^n$ where $C \subset \mathbb{R}^n$ be closed.

Let $u_0 \in \Omega$ and let $u : J \to \Omega$ be an absolutely continuous solution of the problem $(\mathcal{P}_{co})$. Assume that there are two integrable functions $\lambda_1(.)$ and $\lambda_2(.)$ defined on J, satisfying $0 \leq \lambda_1(t) \leq 1 \leq \lambda_2(t)$, $\forall t \in J$ and such that, for almost every $t \in J$, we have

$$-\lambda_1(t)\dot{u}(t) \in Au(t) + F(t, u(t)) \text{ and } -\lambda_2(t)\dot{u}(t) \in Au(t) + F(t, u(t)).$$

Then there exists a nondecreasing absolutely continuous map $t = t(s)$ of the interval J into itself, such that the map $\tilde{u}(s) = u(t(s))$ is a solution of the problem $(\mathcal{P}_F)$.

Theorem 2.

Let $F : J \times \Omega \rightrightarrows \mathbb{R}^n$ be an almost convex closed valued multifunction, satisfying 1., 2. and 3. in Theorem 1. Then,

1. the problem $(\mathcal{P}_F)$ admit at least an absolutely continuous solution in J.

2. for every $\tau \in J$, the attainable set at τ, $A_{u_0}(\tau)$ of the problem $(\mathcal{P}_F)$ coincide avec $A^{co}_{u_0}(\tau)$, the attainable set at τ of the convexified problem $(\mathcal{P}_{co})$.

Keywords: Differential inclusion, almost convex set, time optimal problem.

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Fixed point and global asymptotic stability of nonlinear neutral differential equation

Benhadri Mimia[1]

(1) Faculty of sciences, department of mathematics, University of Skikda, P.O. Box 26, El-Hadaik, Algeria

E-mail: mbenhadri@yahoo.com

Abstract: This talk addresses the stability study for nonlinear neutral differential equations. Thanks to a new technique based on the fixed point theory, we find some new sufficient conditions ensuring the global asymptotic stability of the solution. In particular, the results improve some previous ones in the literature.

In the current talk, we aim at discussing the asymptotic stability in C^1 for a standard form of neutral differential equations as follows,

$$u'(t) = -\sum_{i=1}^{N} a_i\left(t, u_t\right) u(t) + g\left(t, u_t'\right) + f\left(t, u_t\right), t \geq t_0, \tag{1.1}$$

where $f, g \in C\left(\mathbb{R}^+ \times B, \mathbb{R}\right)$ and $a_i \in C\left(\mathbb{R}^+ \times B, \mathbb{R}\right), \left(i = \overline{1, N}\right)$, with

$$B = \left\{\phi \in C\left(\mathbb{R}^-, \mathbb{R}\right) : \phi \text{ bounded}\right\}$$

with the norm $\|\phi\|_\circ := \sup\limits_{\theta \in (-\infty, 0]} |\phi(\theta)|$. Let $u \in C^1(\mathbb{R}, \mathbb{R})$ be bounded and $t \geq 0$ a fixed number, we let $u_t, u_t' \in C$ be defined by $u_t(\theta) = u(t+\theta)$ and $u_t'(\theta) = u'(t+\theta)$ for $\theta \in \mathbb{R}^-$.

Before proceeding, we firstly introduce some assumptions:

(A1) there exists a constant $L > 0$ and a function $b_1 \in C\left(\mathbb{R}, \mathbb{R}^+\right)$ such that, for all $\phi, \psi \in C_L$ and for all $t \geq 0, |f(t, \phi) - f(t, \psi)| \leq |b_1(t)| \|\phi - \psi\|_\circ$.

(A2) there exists a constant $L' > 0$ and a function $b_2 \in C\left(\mathbb{R}, \mathbb{R}^+\right)$ such that, for all $\phi, \psi \in C_{L'}^1$ and for all $t \geq 0$, $|g(t, \phi') - g(t, \psi')| \leq |b_2(t)| \|\phi' - \psi'\|_\circ$.

(A3) $\forall \varepsilon > 0$ and $t_1 \geq 0$, $\exists\ t_2 > t_1$ such that $[t \geq t_2, u_t \in C_L]$, imply $|f(t, u_t)| \leq |b_1(t)| \left(\varepsilon + \|u\|^{[t_1, t]}\right)$.

(A4) $\forall \varepsilon > 0$ and $t_1 \geq 0$, $\exists\ t_3 > t_1$ such that $[t \geq t_3, u' \in C_{L'}^1]$, imply $|g(t, u_t')| \leq |b_2(t)| \left(\varepsilon + \|u'\|^{[t_1, t]}\right)$.

(A5) $\exists\ \alpha_1, \alpha_2 \in C(\mathbb{R}, \mathbb{R})$,(α_2 is bounded) such that $\alpha_1(t) \leq \sum_{i=1}^{N} a_i(t, u_t) \leq \alpha_2(t)$.
(A6) we always assume that $f(t, 0) = g(t, 0) = 0$ for all $t \geq t_0$.

We aim to discuss the asymptotic stability in C^1 for equation (1.1). More precisely, the following result is established:

Theorem. *Assume hypotheses (A1)–(A6) hold, and for any $t \geq t_0$, there exists $\eta \in (0, \frac{1}{2})$ such that*, $\liminf_{t\to\infty} \int_{t_0}^{t} \alpha_1(s)\, ds > -\infty$, *and* $\int_0^t \alpha_1(s) ds \to \infty$ as $t \to \infty$, *and*

$$\int_{t_0}^{t} e^{-\int_s^t \alpha_1(z)dz} \left(|b_1(s)| + |b_2(s)|\right) ds \leq \eta,$$

$$|\alpha_2(t)| \int_{t_0}^{t} e^{-\int_s^t \alpha_1(z)dz} \left(|b_1(s)| + |b_2(s)|\right) ds + \left(|b_1(t)| + |b_2(t)|\right) \leq \eta.$$

Then, the trivial solution to equation (1.1) *is asymptotically stable in C^1.*

Keywords: Fixed point; asymptotic stability; neutral differential equations; variable delays.

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Fixed point and stability of nonlinear problem of delay differential equations

Hocine Gabsi

University of El-Oued, algeria

E-mail: hocinegabsi@gmail.com

Abstract: In this talk we offer existence criteria and sufficient conditions, so that the trivial solution of the system with several delays of feedback control is asymptotically stable. Here the fixed-point technique is a practical method for this purpose. When these results are applied to some special delay mathematical models, some new results are obtained, and many known results are improved.

Keywords: Fixed point, Stability, Variable delays, Nonlinear differential equations.

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Fixed point theorem for nonlinear operator in partially ordered metric spaces and application to ordinary differential equations

Aouine Ahmed Chaouki [(1)]

[(1)] Mohamed-Cherif Messaadia University - Souk Ahas, 41000. Algeria

E-mail: a.aouine@univ-soukahras.dz, chawki81@gmail.com

Abstract: In this talk, we prove a fixed point theorem for rational contraction mappings in complete partially ordered metric spaces. Example is provided to illustrate the validity of our result. Afterwards, we prove existence and uniqueness of the solution of the following periodic boundary value problem

$$\begin{cases} u'(t) = f(t, u(t)) \ if \ t \in I = [0, T], \\ u(0) = u(T), \end{cases} \tag{1}$$

.

where $T > 0$ and $f : I \times \mathbb{R} \to \mathbb{R}$ is a continuous function.
A lower solution for (1) is a function $\alpha \in C^1(I, \mathbb{R})$ such that

$$\begin{cases} \alpha'(t) = f(t, \alpha(t)) fort \in I \\ \alpha(0) < \alpha(T). \end{cases}$$

Fixed point theory fascinated many researchers since 1922 with the famous Banach's fixed point theorem called Banach contraction principle, see [3].
This theorem provided a technique for solving a variety of applied problems in mathematical sciences and engineering. Subsequently, the superb result of Banach was extended and generalized by several authors using various contractive conditions in different spaces.
Existence of a fixed point for contraction type maps in partially ordered metric spaces has been considered recently in [1,4 and 5], where some applications to matrix equation, ordinary differential equations and integral equations are presented.
Inspired by Theorem 6 in [2], it is our purpose in this talk to prove unique fixed point theorems for nonlinear operator in complete partially ordered metric space. Example is furnished to illustrate the validity of our result. Afterwards, we give an application to ordinary differential equations.

Keywords: Fixed point, complete partially ordered metric space, rational contraction type maps, boundary value problems for ordinary differential equations.

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Limit cycle of the discontinous piecewise system formed by cubic Hamiltonian system separated by a straight line

Imane Benabdallah [(1)], Rebiha Benterki [(2)]

[(1,2)]Mathematics Department, Mohamed El Bachir El Ibrahimi University of Bordj Bou Arreridj, Algeria,

E-mail: imane.benabdallah@univ-bba.dz

E-mail: r_benterki@yahoo.fr

Abstract: The periodic orbits play a main role in natural phenomena, as the revolution of the Earth around the Sun, and the moon's rotation around the earth. Due to the existance of periodic orbit in many natural phenomena many authors are interested in the study of periodic solution. So in mathemathics and more precisely in the qualitative theory the study of isolated periodic solution of is the main defficult obdject in the qualitative theory of planar differential systems. We recall that the limit cycle is an isolated periodic solution in the set of all periodic solution of such system.

In 1900, David Hilbert presented 23 mathematical problems at the Paris Conference of the International Congress of Mathematicians. One of them, remains open to this day, is known as 16th Hilbert problem or the extend of 16th Hilbert problem.

The solution of the second part of the sixteenth Hilbert's problem for discontinuous piecewise differential systems have deserved the attention of many researchers. It is a question of finding the upper bound and the possible configuration of the limit cycles for a planar polynomial differential system of degree n. Here we are interested in solving the second part of the sixteenth Hilbert's problem for the discontinuous piecewise differential systems separated by straight line and formed by an arbitrary differential cubic Hamiltonian system with nilpotent centers and an arbitrary differential cubic Hamiltonian system with nilpotent saddles.

Keywords: Discontinous piecewise differential system, limit cycles, straight line, Hamiltonian planar polynomial vector field with linear plus cubic homogeneous terms having a nilpotent center

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Limit cycles for generalized Kukles polynomial differential systems

Amel Boulfoul [(1)],

[(1)] Department of mathematics, 20 august 1955 University. El Hadaiek 21000, Skikda

E-mail: `boulfoul_amel@yahoo.fr`

Abstract: In this talk we consider the limit cycles of a class of polynomial differential Kukles systems of the form

$$\dot{x} = -y, \dot{y} = x - f(x) - g(x)y - h(x)y^2 - l(x)y^3, \tag{9}$$

where $f(x) = \epsilon f_1(x) + \epsilon^2 f_2(x)$, $g(x) = \epsilon g_1(x) + \epsilon^2 g_2(x)$, $h(x) = \epsilon h_1(x) + \epsilon^2 h_2(x)$ and $l(x) = \epsilon l_1(x) + \epsilon^2 l_2(x)$ where $f_k(x)$, $g_k(x)$, $h_k(x)$ and $l_k(x)$ have degree n_1, n_2, n_3 and n_4, respectively for each $k = 1, 2$, and ε is a small parameter. We obtain the maximum number of limit cycles that bifurcate from the periodic orbits of the linear center $\dot{x} = -y$, $\dot{y} = x$ using the averaging theory of first and second order. The main open problem in the qualitative theory of real planar differential systems is the dermination of limit cycles which is related to the second part of the 16th Hilbert problem [3]. The knowledge of the existence or not of periodic solutions is very important for understanding the dynamics of the differential systems. One of good tools for study the periodic solutions is the averaging theory, see for instance Sanders and Verhulst [4] and the references therein.

In different works the limit cycles problem and the center problem for the classical Kukles system

$$\begin{cases} \dot{x} = -y, \\ \dot{y} = x + a_0 y + a_1 x^2 + a_2 xy + a_3 y^2 + a_4 x^3 + a_5 x^2 y + a_6 xy^2 + a_7 y^3. \end{cases}$$

are studied.

Our main results of system (9) are the following ones.

Theorem . For $|\varepsilon| \neq 0$ sufficiently small, the maximum number of limit cycles of the generalized Kukles polynomial differential system (3) bifurcating from the periodic orbits of the linear centre $\dot{x} = -y$, $\dot{y} = x$

1. using the averaging theory of first order is

$$\lambda_1' = \max\left\{\left[\frac{n_2}{2}\right], \left[\frac{n_4}{2}\right] + 1\right\}.$$

2. using the averaging theory of second order is

$$\lambda_2^{'} = \max\left\{\left[\frac{n_2}{2}\right], \left[\frac{n_4}{2}\right]+1, \left[\frac{n_1}{2}\right]+\left[\frac{n_2-1}{2}\right], \left[\frac{n_1}{2}\right]+\left[\frac{n_4-1}{2}\right]+1,\right.$$
$$\left[\frac{n_1-1}{2}\right]+\mu^{'}, \left[\frac{n_2-1}{2}\right]+\left[\frac{n_3}{2}\right]+1, \left[\frac{n_4-1}{2}\right]+\left[\frac{n_3}{2}\right]+2,$$
$$\left.\left[\frac{n_3-1}{2}\right]+\mu^{'}+1\right\},$$

where $\mu^{'} = \min\left\{\left[\frac{n_2}{2}\right], \left[\frac{n_4}{2}\right]+1\right\}$.

In [2] it has been shown that there exists generalized Kukles equation (**??**), having at least $\lambda_2 = \max\left\{\left[\frac{n_1}{2}\right]+\left[\frac{n_2-1}{2}\right], \left[\frac{n_1}{2}\right]+\left[\frac{m}{2}\right]-1, \left[\frac{n_1+1}{2}\right], \left[\frac{n_3+3}{2}\right], \left[\frac{n_3}{2}\right]+\left[\frac{m}{2}\right], \left[\frac{n_2+1}{2}\right]+\left[\frac{n_3}{2}\right], \left[\frac{n_2}{2}\right], \left[\frac{m-1}{2}\right], \left[\frac{n_1-1}{2}\right]+\mu, \left[\frac{n_3+1}{2}\right]+\mu, 1\right\}$ limit cycles. The result in Theorem 2 improves this lower estimate $\left(\lambda_2^{'} > \lambda_2 \text{ for all } n_1 \geq 1, n_2 \geq 1, n_3 \geq 1,\ m \geq 2 \text{ and } n_4 \geq \max\{3, n_2, m-1\}\right)$. For each fixed $n_1 \geq 1$, $n_2 \geq 1$ $n_3 \geq 1$ and $m \geq 2$ there exists $n_4^{'} \geq \max\{3, n_2, m-1\}$ such that $\lambda_2^{'} > \lambda_2$ for all $n_4 \geq n_4^{'}$.

Keywords: Limit cycle, Averaging theory, Kukles systems.

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Monotone positive solution of fourth order boundary value problem with mixed integral and multi-point boundary conditions

Houari Nourredine [(1)], Haddouchi Faouzi [(2)]

[(1)] Laboratory of fundamental and Applied Mathematics of Oran, Department of Mathematics, University of Oran 1, Oran, Algeria
E-mail: `noureddinehouari.mi@gmail.com`

[(2)] Department of Physics, University of sciences and technology of Oran-MB, Oran, Algeria
Laboratory of fundamental and Applied Mathematics of Oran, Department of Mathematics, University of Oran 1, Oran, Algeria
E-mail: `fhaddouchi@gmail.com`

Abstract: In this talk, we study the existence of monotone positive solution for the following nonlinear fourth order boundary value problem with mixed integral and multi-point boundary conditions

$$u''''(t) + f(t, u(t), u'(t)) = 0,\ t \in (0, 1), \tag{10}$$

$$u'(0) = u'(1) = u''(0) = 0,\ u(0) = \alpha \int_{\nu}^{\xi} u(s)ds + \sum_{i=1}^{n} \beta_i u'(\eta_i), \tag{11}$$

where

H1) $f \in C([0, 1] \times [0, \infty) \times [0, \infty), [0, \infty))$.

H2) $\alpha \geq 0; \beta_i \geq 0$, and $0 \leq \nu < \eta_1 < \eta_2 < ... < \eta_n < \xi \leq 1, 1 \leq i \leq n$.

Our main tool is a fixed point theorem in a cone.

Keywords: Existence, monotone positive solution, fixed point theorem, boundary value problem, cone.

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Numerical solution of nonlinear integral equations via collocation method based on cardinal functions

Rebiha Zeghdane [(1)]

[(1)] Department of Mathematics, Bordj Bou-Arreridj university, Algeria
E-mail: rebihae@yahoo.fr

Abstract: In the last decades, there has been an increasing interest in applying cardinal basis functions [1] for various types of problems. Spectral methods [2, 3] have been extensively used to find approximate solutions of various types of linear and nonlinear equations such as differential equations and integral equations. They have a wide range of applications in science and engineering. Numerical methods are important tools for calculating approximation solutions of stochastic differential equations. In recent years many numerical methods for deterministic and stochastic integral equations have been designed. Noting that finding the exact solutions for most of these equations is hard, therefore, we have to apply approximate numerical methods to obtain numerical solutions. In this work, we give a new numerical technique for solving stochastic integral equations. A new operational matrix for integration of cardinal Legendre polynomials are introduced. By using this new operational matrix of integration and the so-called collocation method, stochastic nonlinear integral equations are reduced to systems of algebraic equations with unknown coefficients. Only small dimension of Legnedre operational matrix is needed to obtain a satisfactory result. Some error estimations are provided and the results of numerical experiments are compared with the analytical solution in illustrative examples to confirm the accuracy and efficiency of the presented method.

In recent years many numerical methods for deterministic and stochastic integral equations have been designed, for example, Adomian method, implicit Taylor methods and recently the operational matrices of integration for orthogonal polynomials, Legendre wavelets, Chebychev polynomials,..etc. Several analytical and numerical methods have been proposed for solving various types of stochastic problems with the classical Brownian motion. Noting that finding the exact solutions for most of these equations is hard, therefore, we have to apply approximate numerical methods to obtain numerical solutions. The main characteristic of the approach using this technique is that it reduces these problems to a systems of algebraic equations which simplifying the problem. In recent years, Cardinal

functions have been finding an important role in numerical analysis, in particulary for solving integral equations In this talk, we use cardinal Legendre function to find numerical solution of the following stochastic integral equations.

$$X(t) = X_0 + \int_0^t a(s, X(s))ds + \int_0^t b(s, X(s))dB(s), \tag{12}$$

under the initial condition $X(0) = X_0$, where $X(t)$ is an unknown process, which shoud be computed. for $0 \leq t, s \leq 1$, X_0 is a random variable, $B(s)$ is a Brownian motion and where $a(s, X(s, \omega))$, $b(s, X(s, \omega))$ for $s, t \in [0, 1]$ are known stochastic processes defined on the same filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$ with natural filtration $\mathcal{F}_t$, X_0 is the known random variable with $E|X_0|^2 < +\infty$ and $X(t)$ is unknown stochastic process. The second integral in (12) is the Ito integral. Furthermore, all Lebesgue's and Ito integrals in (12) are well defined. Note that the existence and the uniqueness of a solution for the problem (12) are investigated in [4].

Keywords: Legendre polynomials, Stochastic differential equation, Spectral method, Brownian motion, Collocation method, Convergence analysis.

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On the first integral for a class of Kolmogorov systems

Rachid Boukoucha [(1)]

[(1)] Lab. de Mathématiques Appliquées, Université de Bejaia,
06000 Bejaia, Algérie.
E-mail: rachid.boukoucha@univ-bejaia.dz

Abstract: Many mathematical models in biology science and population dynamics, frequently involve the systems of ordinary differential equations. Kolmogorov models are widely used in ecology to describe the interaction between two populations, and a limit cycle corresponds to an equilibrium state of the system. There are many natural phenomena which can be modeled by the Kolmogorov systems such as mathematical ecology and population dynamics, chemical reactions, plasma physics, hydrodynamics, economics, etc...

In this talk we charaterize the integrability and the non-existence of limit cycles of Kolmogorov systems of the form

$$\begin{cases} x' = x\left(P(x,y) + \left(\frac{R(x,y)}{S(x,y)}\right)^{\lambda}\right), \\ y' = y\left(Q(x,y) + \left(\frac{R(x,y)}{S(x,y)}\right)^{\lambda}\right), \end{cases} \tag{13}$$

where $P(x,y)$, $Q(x,y)$, $R(x,y)$, $S(x,y)$ are homogeneous polynomials of degree n, n, m, a respectively and $\lambda \in \mathbb{Q}^*$.

Our main result on the Kolmogorov system (13) is the following.

Theorem . Consider a Komogorov system (13), then the following statements hold.

(a) If $(Q(\cos\theta,\sin\theta) - P(\cos\theta,\sin\theta))\cos\theta\sin\theta \neq 0$, $R(\cos\theta,\sin\theta)\,S(\cos\theta,\sin\theta) \geq 0$, $S(\cos\theta,\sin\theta) \neq 0$ for $\theta \in \left(0,\frac{\pi}{2}\right)$ and $\lambda m - \lambda a \neq n$, then system (13) is integrable.

Moreover, the system (13) has no limit cycle.

(b) If $(Q(\cos\theta,\sin\theta) - P(\cos\theta,\sin\theta))\cos\theta\sin\theta \neq 0$, $S(\cos\theta,\sin\theta) \neq 0$, $R(\cos\theta,\sin\theta)\,S(\cos\theta,\sin\theta) \geq 0$ for $\theta \in \left(0,\frac{\pi}{2}\right)$ and $\lambda m - \lambda a = n$, then system (13) is integrable.

Moreover, the system (13) has no limit cycle.

(c) If $(Q(\cos\theta, \sin\theta) - P(\cos\theta, \sin\theta))\cos\theta\sin\theta = 0$ for all $\theta \in \mathbb{R}$, then system (13) has the first integral $H(x, y) = \frac{y}{x}$.

Moreover, the system (13) has no limit cycle.

Keywords: Kolmogorov system, first integral, periodic orbits, limit cycle.

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Some Results for a Nonlinear Fourth-Order Boundary Value Problem with Integral Boundary Condition

GUENDOUZ Cheikh

Laboratory of Fundamental and Applied Mathematics of Oran(LMFAO). University of Oran1. Oran, Algeria.

E-mail: `guendouzmath@yahoo.fr`

Abstract: This talk is concerned with the existence of at least one positive monotone solution for a class of nonlinear fourth order boundary value problems with integral boundary condition. Our analysis relies on Leary-Shauder fixed point theorem and properties of Green's function. Two examples are presented to illustrate our theoretical results. Differential equations appear in almost all areas of the science and technology: Mathematics, physics, viscoelasticity, chemistry, biology, engineering, mechanics, and economics. In practice, only positive solutions can be useful because they correspond to measurable parameters such as temperature, density ..., parameters that are used in different laws of physics. The resolution of differential equations or even boundary problems associated with differential equations, is a very large field investigation. Recently, the study of existence of positive solution to fourth-order boundary value problems has gained much attention and is rapidly growing field. In this talk, we are concerned with the following fourth-order boundary value problem having an integral boundary condition

$$u^{(4)}(t) + f(t, u(t), u'(t)) = 0,\ t \in (0,1), \tag{14}$$

$$u'(0) = u'(1) = u''(0) = 0,\ u(0) = \int_0^1 g(s)u'(s)ds, \tag{15}$$

where

(C_1) $f \in C([0,1] \times [0,+\infty) \times [0,+\infty), [0,+\infty))$,

(C_2) $g \in C([0,1], [0,\infty))$.

For convenience, we denote $\alpha = \int_0^1 g(t)dt,\ \beta = \int_\theta^{1-\theta} g(t)dt$.

At first, we consider the Banach space $X = C^1([0,1], \mathbb{R})$ equipped with the norm

$$\|u\| = \max\Big\{||u||_\infty, ||u'||_\infty\Big\}.$$

Let $\theta \in (0, \frac{1}{2})$ be fixed. Define the cone $K \subset X$

$$K = \left\{ u \in X,\ u(t) \geq 0, u'(t) \geq 0, t \in [0,1] : \min_{t \in [\theta, 1-\theta]} u(t) \geq \frac{\theta^3}{6} \frac{(1+\beta)}{(1+\alpha)} \|u\| \right\},$$

Let $A : K \to X$ the operator defined as $\qquad A : K \to C[0,1]$

$$Au(t) = \int_0^1 T(t,s) f(s, u(s), u'(s)) ds, \quad t \in [0,1]. \tag{16}$$

Lemma The operator A defined by (16) is completely continuous and $A(K) \subset K$.

Notation: For convenience, we introduce the following notations $f^\delta = \lim_{x+y \to \delta} \left\{ \max_{0 \leq t \leq 1} \frac{f(t,x,y)}{x+y} \right\}$, $\Lambda_1 = 1 + \alpha$, $\Lambda_2 = \frac{\Lambda_1^{-1}}{2}$, where δ denotes either 0 or ∞.

Theorem 1.
If $0 \leq f^0 < \Lambda_2$, then boundary value problem (14)-(15) has at least one monotone positive solution in K.

Theorem 2.
If $0 \leq f^\infty < \Lambda_2$, then the problem (14)-(15) has at least one monotone positive solution.

Keywords: Monotone positive solutions, Leray-Schauder's fixed point theorem, fourth-order integral boundary value problems, existence.

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Partial Differential Equations

Analytic semigroups generated by the dispersal process in two habitats incorporating individual behavior at the interface

Abdallah Menad [(1)], Angelo Favini [(2)], Rabah Labbas [(3)], Ahmed Medeghri[(4)]

[(1)] Laboratoire de Mathématiques Pures et Appliquées, Université Abdelhamid Ibn Badis, 27000 Mostaganem, Algeria E-mail: `abellah.menad@univ-mosta.dz`

[(2)] Università degli Studi di Bologna, Dipartimento di Matematica, Piazza di Porta S. Donato, 5, 40126 Bologna, Italy
E-mail: `angelo.favini@unibo.it`

[(3)]LMAH, Normandie Univ., UNIHAVRE, LMAH, FR-CNRS-3335, 76600 Le Havre, France
E-mail: `rabah.labbas@univ-lehavre.fr`

[(4)] Laboratoire de Mathématiques Pures et Appliquées, Université Abdelhamid Ibn Badis, 27000 Mostaganem, Algeria E-mail: `ahmed.medeghri@univ-mosta.dz`

Abstract: In this talk we study an elliptic differential equation set in two habitats with skewness boundary conditions at the interface. This problem represents the linear stationary case of dispersal problems of population dynamics which incorporate responses at interfaces between the habitats.
We prove that this operator generates an analytic semigroup in an adapted space of Hölder-continuous functions.
Our goal in this talk is to analyze the analogous situation as [2] in two dimension space. More precisely, we will be concerned with the study of the analyticity of the C_0-semigroup generated by the dispersal process in two habitats under some skewness condition and continuous dispersal condition at the interface which represent the behavior of the individuals at boundaries. Our problem is reduced to an operational form of the type:

$$\begin{cases} u''(x) - B^2 u(x) = G(x) \quad \text{on} \quad]-l, 0[\cup]0, L[\\ u_-(-l) = 0 \text{ and } u'_+(L) = 0 \\ \begin{cases} d_- \left[u''_-(0^-) + Au_-(0^-)\right] - r_- u_-(0^-) \\ = d_+ \left[u''_+(0^+) + Au_+(0^+)\right] + r_+ u_+(0^+) \end{cases} \\ (1-p) d_- u'_-(0^-) = p d_+ u'_+(0^+). \end{cases}$$

The operator A verifies the following ellipticity hypothesis:

$$\rho(A) \supset [0, +\infty[\text{ et } \exists C > 0 : \forall \lambda \geq 0, \left\|(A - \lambda I)^{-1}\right\|_{L(E)} \leq \frac{C}{1 + |\lambda|}.$$

Where $\rho(A)$ denotes the resolvent set of A.

The operator $B := -(-A)^{\frac{1}{2}}$ generates an analytic semigroup which makes it possible to find a representation of the solution.

Keywords: Semigroups ; population dynamics ; elliptic differential equation.

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BLOW UP OF SOLUTIONS FOR A HYPERBOLIC-TYPE EQUATION WITH TIME DELAY

Hazal Yüksekkaya [(1)], Erhan Pişkin [(2)]

[(1)] Department of Mathematics, University of Dicle, Diyarbakır, Turkey
E-mail: hazally.kaya@gmail.com

[(2)] Department of Mathematics, University of Dicle, Diyarbakır, Turkey
E-mail: episkin@dicle.edu.tr

Abstract: The first equations with delay were studied by brothers Bernoulli and Leonard Euler in the eighteenth century. Systematical study started at the 1940s by A. Myshkis and R. Bellman. Since 1960 there have been appeared many surveys on the subject. Robust control of systems with uncertain delay was started in the middle of 1990s and led to the "delay bloom" in the begining of the twenty-first century.Time-delay systems are also called systems with aftereffect or dead-time, hereditary systems, equations with deviating argument, or differential-difference equations. They belong to the class of functional differential equations which are infinite-dimensional, as opposed to ordinary differential equations. Time-delay often appears in many control systems (such as aircraft, chemical or process control systems, and communication networks), either in the state, the control input, or the measurements. There can be transport, communication, or measurement delays. Controlling the behavior of solutions for partial differential equations with time delay effects has become an active research area. Generally, delay effects occur in many applications and practical problems such as physical, chemical, biological, thermal and economics. In many cases, delay is a source of instability, even an arbitrarily small delay may destabilize a system which is uniformly asymptotically stable in the absence of delay unless additional conditions or control terms have been used. In this work, we consider a hyperbolic-type equation with time delay. Under appropriate conditions, we establish the blow up of solutions.

Keywords: Blow up, Hyperbolic-type equation, Time delay.

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Blow-up of solutions for a problem with Balakrishnan-Taylor damping and nonlocal singular viscoelastic equations

Draifia Ala Eddine [(1)]

[(1)] Laboratory of Mathematics, Informatics and Systems (LAMIS), Larbi Tebessi University, 12002 Tebessa, Algeria

E-mail: draifia1991@gmail.com

Abstract: In fluid dynamics, the blow-up problem of solutions has attracted much attention and challenge among physicists and mathematicians. In this talk, we study a singular nonlinear one-dimensional viscoelastic nonlocal problem with Balakrishnan-Taylor damping terms and logarithmic nonlinearity source. We demonstrate that the logarithmic nonlinearity source of polynomial type is able to force solutions to blow up infinite time even in presence of stronger damping with non positive initial energy combined with a positive initial energy. More precisely, we study blow-up of solutions of the following problem:

$$u_{tt}(t) - M(t)\frac{1}{x}(xu_x(t))_x + \int_0^t g(t-s)\frac{1}{x}(xu_x(x,s))_x\,ds + au_t(t) = u(t)\,|u(t)|^{p-2}, \text{ in } Q, \tag{17}$$

$$\begin{cases} u(x,0) = u_0(x),\ u_t(x,0) = u_1(x),\ x \in (0,\alpha), \\ u_x(\alpha,t) = 0,\ \int_0^\alpha xu(x,t)\,dx = 0 \quad t \in [0,T], \end{cases} \tag{18}$$

where $Q := (0,\alpha)\times(0,T)$, $\alpha < \infty$, $T < \infty$, $p > 4$, $g(.) : \mathbb{R}^+ \to \mathbb{R}^+$ are given functions which will be specified later, and $M(t) := \xi_0 + \xi_1 \|u_x(t)\|^2_{L^2_x(0,\alpha)} + \sigma(u_x(t), u_{xt}(t))_{L^2_x(0,\alpha)}$, where u is the plate transverse displacement, x is the spatial coordinate in the direction of the fluid flow, and t is time. The viscoelastic structural damping terms are denoted by ξ_1, σ is the nonlinear stiffness of the membrane, and ξ_0 is an in-plane tensile load. All quantities are physically non-dimensional zed and ξ_0, ξ_1 and σ are fixed positive.

Definition 1. A solution u of $(1)-(2)$ is called blow-up if there exists a finite time T^* such that

$$\lim_{t\to T^{*-}} \left(\|u_x(t)\|^2_{L^2_x(0,\alpha)}\right)^{-1} = 0.$$

Further assumptions on g which ensure the Blow-up phenomena of system $(1)-(2)$ are

$$(\mathbf{A}) \quad \begin{cases} g(s) \geq 0, \ g'(s) \leq 0 \text{ and} \\ \int_0^\infty g(s)\, ds < \dfrac{p(p-2)}{(p-1)^2}\xi_0 < \xi_0, \quad \text{for } p > 4. \end{cases}$$

Keywords: Balakrishnan–Taylor damping; Viscoelastic equations; blow-up.

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Elliptic p-Kirchhoff type systems with critical Sobolev exponent in $\mathbb{R}^N$

Naima Keddar
Higher School of Management of Tlemcen. Algeria
E-mail: `naima_keddar@yahoo.fr`

Abstract: In this talk, we are concerned to study the existence and non-existence of solutions to the following Kirchhoff-type systems involving the critical Sobolev exponent

$$(P)\begin{cases} -(a_1+b_1\|u\|^p)\left[\mathrm{d}iv\left(|\nabla u|^{p-2}\nabla u\right)\right]=\dfrac{2q}{q+q'}|u|^{q-2}u\ \ |v|^{q'}+\lambda_1 f(x), \\ -(a_2+b_2\|v\|^p)\left[\mathrm{d}iv\left(|\nabla v|^{p-2}\nabla v\right)\right]=\dfrac{2q'}{q+q'}|u|^{q}\,|v|^{q'-2}\,v+\lambda_2 g(x), \\ (u,v)\in W^{1,p}\left(\mathbb{R}^N\right)\times W^{1,p}\left(\mathbb{R}^N\right) \end{cases} \text{ in } \mathbb{R}^N$$

where $1<p<N,\ a_1,\ a_2\geq 0,\ b_1,\ b_2>0,\ q,\ q'>1,\ q+q'=p^*,\ p^*=pN/[N-p]$ is the critical Sobolev exponent, $\lambda_1,\ \lambda_2>0$ are a parameters, $f,\ g\in W^*\setminus\{0\}$.

Let the positive constant

$$S_{q,q'}:=\inf_{\substack{(u,v)\in W^{1,p}\left(\mathbb{R}^N\right)\times W^{1,p}\left(\mathbb{R}^N\right)\\(u,v)\neq(0,0)}}\frac{\|u\|^p+\|v\|^p}{\left(\int_{\mathbb{R}^N}|u|^q\,|v|^{q'}\,dx\right)^{p/p^*}}$$

First we introduced some assumptions which we need to prove our results

(H_1) $p^*=2p,\ a_2=a_2=0,\ b_1,\ b_2>S_{q,q'}^{-2}$.

(H_2) $p^*=2p,\ b_1,\ b_2\geq S_{q,q'}^{-2},\ a_1,\ a_2>0$.

(H_3) $p^*>2p,\ a>0,\ b>\frac{p^*-p}{p}\left(2\frac{2p-p^*}{pa}\right)^{\frac{2p-p^*}{p^*-p}}2^{\frac{p}{p^*-p}}\left(S_{q,q'}\right)^{-\frac{p^*}{p^*-p}}$.

(H_4) $p^*=2p,\ a_2=a_2=0,\ b_1,b_2>S_{q,q'}^{-2}$.

(H_5) $p^*=2p,\ b_1,b_2\geq S_{q,q'}^{-2},\ a_1,a_2>0$.

The main results in this paper are the following:

Theorem 1.[Non-existence Result] Suppose that $(\lambda_1,\lambda_2)=(0,0)$ and assume (H_1) or (H_2) or (H_3).Then the problem (P) has no non-trivial solution.

Theorem 2.[Existence of a critical point with negative energy] Suppose that $f,g\in W^*\left(\mathbb{R}^N\right)\setminus\{0\}$, and assume (H_4) or (H_5) then there exists a constants $\lambda_1^*,\lambda_2^*,\lambda_3^*>0$ such

that for any λ_1, λ_2 verified (19), system (P) has a solution (u_1, v_1) with negative energy.

$$\begin{cases} \lambda_1 \leq \lambda_1^* & \text{if } \lambda_1 \neq 0 \text{ and } \lambda_2 = 0 \\ \lambda_2 \leq \lambda_2^* & \text{if } \lambda_1 = 0 \text{ and } \lambda_2 \neq 0 \\ \min(\lambda_1, \lambda_2) \leq \lambda_3^* & \text{if } \lambda_1 \neq 0 \text{ and } \lambda_2 \neq 0 \end{cases} \tag{19}$$

Keywords: Variational methods, Mountain Pass Theorem, Ekeland Variational Principle, critical exponent of Sobolev, Kirchhoff problems.

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Energy estimates and boundary observability for the wave equation in some time-varying domains

Seyf Eddine Ghenimi [(1)], Abdelmouhcene Sengouga [(2)]

[(1)] [(2)] Laboratory of Functional Analysis and Geometry of Spaces,
University Mohamed Boudiaf, M'sila, Algeria.
E-mail: `seyfeddine.ghenimi@univ-msila.dz` [(1)]
E-mail: `abdelmouhcene.sengouga@univ-msila.dz` [(2)]

Abstract: The purpose of this presentation is to consider a wave equation in an interval with two ends moving of fixed length L and traveling with constant speed v is strictly less then the characteristic speed of the wave, i.e.

$$0 < v < 1. \tag{20}$$

For $L > 0$ and $T > 0$, we denote the interval

$$\mathbf{I}_t := (vt, L + vt), \quad \text{for } t \in (0, T).$$

Let us now consider the following wave equation with homogeneous Dirichlet boundary conditions

$$\begin{cases} \phi_{tt} - \phi_{xx} = 0, & \text{for } x \in \mathbf{I}_t, t \in (0, T), \\ \phi(vt, t) = \phi(L + vt, t) = 0, & \text{for } t \in (0, T), \\ \phi(x, 0) = \phi^0(x), \quad \phi_t(x, 0) = \phi^1(x), & \text{for } x \in \mathbf{I}_0, \end{cases} \tag{WP}$$

Under the assumption (20), we already know that for every initial data

$$\phi^0 \in H_0^1(\mathbf{I}_0), \quad \phi^1 \in L^2(\mathbf{I}_0), \tag{21}$$

the solution of Problem (WP) exists and satisfies

$$\phi \in C\left([0, T]; H_0^1(\mathbf{I}_t)\right) \quad \text{and} \quad \phi_t \in C\left([0, T]; L^2(\mathbf{I}_t)\right). \tag{22}$$

The exact solution of the Problem is given by a series formulas

$$\phi(x, t) = \sum_{n \in \mathbb{Z}^*} c_n \left(e^{n\pi i(1-v)(t+x)/L} - e^{n\pi i(1+v)(t-x)/L}\right), \text{ for } x \in \mathbf{I}_t \text{ and } t \in (0, T), \tag{23}$$

where the coefficients $c_n \in \mathbb{C}$ are given in function of the initial data (21). Then, we show that the series formulas (WP) can be manipulated to establish the following results:

- The Energy $E_v(t)$ of the solution Problem (WP) , given by

$$E_v(t) = \frac{1}{2}\int_{vt}^{L+vt} \phi_x^2(x,t) + \phi_t^2(x,t)\, dx, \qquad \text{for } t \geq 0. \tag{24}$$

  is periodic of period

$$T_v := \frac{2L}{1-v^2}$$

  and remains bounded in time under the assumption (20).

- The wave equation (WP) is exactly observable at any endpoint $x = x_b + vt$, where $x_b = 0$ or $x_b = L$. The time of observability is exactly equal to T_v. The observability constants are explicitly given in function of L and v.

- Using the Hilbert uniqueness method (HUM), the above observability results implies controllability result at each one endpoints.

Keywords: Wave equation; time-varying domains; energy estimates; boundary observability; Hilbert Uniqueness Method.

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Existence and blow-up of result to the Cauchy problem for a wave p-Laplace equation with source term in $\mathbb{R}^n$

Abderrahmane Beniani [(1)], Khaled Zennir [(2)]

[(1)] University of Ain Temouchent, Laboratory of Analysis and Control of Partial Differential Equations, Ain Temouchent 46000, Algeria.
E-mail: `a.beniani@yahoo.fr`

[(2)] Department of Mathematics, College of Sciences and Arts, Qassim University, Ar-Rass, Saudi Arabia.
E-mail: `k.Zennir@qu.edu.sa`

Abstract: In this talk, we consider an initial-boundary value problem for the nonlinear wave equation containing the $p-$Laplacian operator,

$$u_{tt} - \phi(x)\mathrm{div}\left(|\nabla u|^{p-2}\nabla u\right) + \mu u_t = b|u|^{m-2}u.$$

Under suitable conditions on the initial datum, we prove, by using Faedo-Galerkin method, the existence of local weak solutions, which can be extended globally provided the weight of $p-$Laplacian operator dominates the source in an appropriate sense and establish a polynomial decay result. Moreover, a blow-up result is proved for solutions with negative initial total energy.

In this talk, we investigate a global studies related to global existence and asymptotic behavior properties of solutions for initial boundary value problem of wave equation containing the $p-$Laplacian operator

$$\begin{cases} u_{tt} - \phi(x)\mathrm{div}\left(|\nabla u|^{p-2}\nabla u\right) + \mu u_t = b|u|^{m-2}u & \text{in } \mathbb{R}^n \times \mathbb{R}^+ \\ u(x,0) = u_0(x) \in \mathcal{D}^{1,p}(\mathbb{R}^n), \quad u_t(x,0) = u_1(x) \in L^2_\rho(\mathbb{R}^n), \end{cases} \tag{25}$$

where the spaces $\mathcal{D}^{1,p}(\mathbb{R}^n), L^2_\rho(\mathbb{R}^n)$ and $\phi(x) > 0, \forall x \in \mathbb{R}^n, (\phi(x))^{-1} = \rho(x)$. The function $\rho : \mathbb{R}^n \to \mathbb{R}^*_+, \rho(x) \in C^{0,\gamma}(\mathbb{R}^n)$ with $\gamma \in (0,1)$ and $\rho \in L^{n/2}(\mathbb{R}^n) \cap L^\infty(\mathbb{R}^n)$.

Keywords: Generalized Sobolev spaces, Viscoelastic wave equation, Unbounded domains, Blow up, General decay.

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Existence and lower bounds for the blow-up time in logarithmic wave equation with nonlinear dynamical boundary conditions

Nazlı Irkıl [(1)], Erhan Pişkin [(2)]

[(1)] Dicle University, Department of Mathematics, Diyarbakır, Turkey
E-mail: nazliirkil@gmail.com

[(2)] Dicle University, Department of Mathematics, Diyarbakır, Turkey
E-mail: episkin@dicle.edu.tr

Abstract: In this presantation, the initial boundary value problem of linear wave equation with dynamic boundary condition which is including logarithmic source term and nonlinear damping term have been investigated. Dynamic boundary problems are very natural in many mathematical models, such as hydrologic filtration process, heat transfer between solid and moving fluid, thermoelasticity, diffusion phenomenon, hydrodynamics (see [1,5]). Many authors gave big attention to this problem in the absence of logarithmic source term for quite a long time. They made a lot of progress, as reported in [2, 4, 6,10] with references therein. Lately, wave eqaution with logarithmic source term which is applied in many branches of physics was discussed by many author (see [3, 7, 8, 9]). Consequently, by motivated this work we examine the effect of logarithmic nonlinearity to dynamic boundary condition. We study local existence result by using Schauder fixed point theorem. Under suitable assumptions on initial data, the lower bound time of blow up result is investigated. These results fill in the gaps in previous studies on this type of models.
Keywords: Blow up, Existence, Lower bound, Logarithmic source term

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General stability result for an abstract viscoelastic equation with time delay

Houria Chellaoua [(1)], Yamna Boukhatem [(2)]

[(1)] Laboratory of Pure and Applied Mathematics, University of Laghouat, P.O. BOX 37G, Laghouat (03000), Algeria.
E-mail: `chellaoua.houria@univ-ghardaia.dz`

[(2)] Laboratory of Pure and Applied Mathematics, University of Laghouat, P.O. BOX 37G, Laghouat (03000), Algeria.
E-mail: `y.boukhatem@lagh-univ.dz`

Abstract: In this talk, we consider an abstract viscoelastic equation with time delay and a nonlinear source term. Let H be a real Hilbert space with inner product and related norm denoted by $\langle .,. \rangle$ and $\| \, . \, \|$, respectively. Let $A : D(A) \longrightarrow H$ and $B : D(B) \longrightarrow H$ be self-adjoint linear positive operators with domains $D(A) \subset D(B) \subset H$ such that the embeddings are dense and compact. $h : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is the kernel of the memory term, $\tau > 0$ represents a time delay and $F \ : D(A^{\frac{1}{2}}) \to H$ is function satisfying some conditions to be specified later. We consider the following abstract evolution equation

$$\begin{cases} u_{tt}(t) + Au(t) - \int_0^t h(t-s)Bu(s)ds + \mu_1 u_t(t) + \mu_2 u_t(t-\tau) = F(u(t)), & t \in (0, +\infty), \\ u_t(t-\tau) = f_0(t-\tau) & t \in (0, \tau), \\ u(0) = u_0, \quad u_t(0) = u_1, \end{cases} \tag{26}$$

where the initial datum (u_0, u_1, f_0) belongs to suitable spaces, μ_1 is a positive constant and μ_2 is a real number such that $|\mu_2| \leq \mu_1$. We are interested in giving optimal, explicit and general decay rates of solution of problem (26) under some suitable assumptions. More precisely, we are intending to extend the results of Messaoudi 5 and Mustafa 6 to the abstract viscoelastic equation with time delay in Hilbert spaces; the system (26). is strictly increasing and strictly convex C^2 function on $(0, r]$, $r \leq h(0)$, with $G(0) = G'(0) = 0$, such that

$$h'(t) \leq -\zeta(t)G(h(t)), \quad \forall t \geq 0, \tag{27}$$

where $\zeta : \mathbb{R}_+ \to \mathbb{R}_+$ is a nonincreasing differentiable function, we establish explicit and general decay rate results of the energy by introducing a suitable Lyaponov functional and

some proprieties of the convex functions under suitable conditions. The energy functional E associated with problem (26) defined by

$$\begin{aligned} E(t) &= \frac{1}{2}\left(\left\|A^{\frac{1}{2}}u\right\|^2 - \int_0^t h(s)ds\left\|B^{\frac{1}{2}}u\right\|^2 + \|u_t\|^2 + (h\diamond B^{\frac{1}{2}}u)(t)\right) \\ &\quad -\mathcal{F}(u) + \frac{\xi\tau}{2}\int_0^1 \|z(\rho,t)\|^2\, d\rho, \qquad \forall t\in\mathbb{R}_+, \end{aligned} \tag{28}$$

where

$$(h\diamond B^{\frac{1}{2}}u)(t) = \int_0^t h(t-s)\left\|B^{\frac{1}{2}}u(t) - B^{\frac{1}{2}}u(s)\right\|^2 ds \tag{29}$$

and ξ a positive constant and we finish by some applications to illustrate our results. This work generalizes the previous results without time delay term to those with delay.

Keywords: Abstract viscoelastic equation, general decay, nonlinear source term, time delay.

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Identifying unknown boundary in a pollution problem: A sentinel method

ELHAMZA Billal [(1)], HAFDALLAH Abdelhak [(2)]

[(1)],[(2)] Department of Mathematics and Computer Sciences, Larbi Tbessi University, Laboratory of Mathematics, Informatics and Systems (LAMIS), Constantine Street, Tebessa, 12002, State, Algeria.

E-mail: `bilal.elhamza@univ-tebessa.dz`

E-mail: `abdelhak.hafdallah@univ-tebessa.dz`

Abstract: In this talk, we identify unknown boundary in a linear parabolic equation. The main tool used here is a method introduced by J. L. Lions called sentinel method. We prove that the existence of the sentinel is equivalent to solve a null controllabilty problem by using the Carleman inequality, which is an observability inequality and is originally an essential tool for studying the unique continuity of solutions to partial differential equations.

Let $\Omega \subset \mathbb{R}^n$ be an open bounded domain, its boundary Γ be of class C^2. We denote $\Omega \times (0,T)$ by Q and $\Gamma \times [0,T]$ for fixed time $T > 0$ by Σ.

We consider the following system modeling a problem of pollution

$$\left\{\begin{array}{rcccc} \frac{\partial z}{\partial t} - \Delta z + p_0(x) z & = & 0 & \text{in} & \Omega \times (0,T)\,, \\ z\,(x,T) & = & y_0 + \tau \widehat{y}_0 & \text{in} & \Omega, \\ z\,(x,t) & = & \left\{\begin{array}{cc} \sigma + \sum_1^m \lambda_i \widehat{\sigma}_0 & \text{on} \quad \Gamma_0 \times (0,T) \\ 0 & \text{on} \quad \Gamma \diagdown \Gamma_0 \times (0,T)\,. \end{array}\right. \end{array}\right. \tag{30}$$

where

- The boundary condition is unknown on a part $\Gamma_0 \times (0,T)$, its structure is $\sigma + \sum_1^m \lambda_i \widehat{\sigma}_0$, with σ and $\widehat{\sigma}_0$ are known whereas λ_i are unknown.
- The initial condition is also partially unknown where y_0 is known and $\tau \widehat{y}_0$ is unknown.

we are interested in identifying the parameters λ_i without any attempt to calculate $\tau \widehat{y}_0$ in the system (30) which models a problem of pollution from observing the flux of the data $\frac{\partial y}{\partial \nu}$ on a part $O \in L^2(\Gamma \times (0,T))$.

For this aim, we use the theory of sentinel which lies on three considerations: - A state equation: the solution of the problem (30). - An observation: $\frac{\partial y}{\partial \nu} = m_0$. - A functional S defined for $h_0 \in L^2(O \times (0,T))$ as follow

$$S(\lambda, \tau) = \int_0^T \int_O (h_0 + u) \frac{\partial y}{\partial \nu} d\Gamma dt. \tag{31}$$

We define the adjoint state as follow

$$\begin{cases} -\frac{\partial y}{\partial t} - \Delta y + p_0(x)y &= 0 & \text{in} & \Omega \times (0,T)\,, \\ y\,(x,0) &= 0 & \text{in} & \Omega, \\ y\,(x,t) &= h_0\chi_O + w\chi_\omega & \text{on} & \Gamma \times (0,T)\,, \end{cases} \tag{32}$$

which satisfies.

$$y(0) = 0. \tag{33}$$

Let $Y^\perp$ the orthogonale of Y in $L^2(\omega \times (0,T))$, and let also $Y_\theta = \frac{1}{\theta}Y$, where θ is positive function precisely defined later.

Proposition: The existence of the sentinel (2) holds if and only if the null boundary controllability problem (32) which satisfies (4) has a solution.

Theorem: Given $h_0 \in L^2(O \times (0,T))$, $w_0 \in Y_\theta$, and $p_0 \in L^\infty(Q)$. Then, there exists a control functon $v \in L^2(\omega \times (0,T))$, and

$$v \in Y^\perp \tag{34}$$

such that the solution $q = q(x,t)$ of the following problem

$$\begin{cases} -\frac{\partial y}{\partial t} - \Delta y + p_0(x)y &= 0 & \text{in} & \Omega \times (0,T)\,, \\ y\,(x,0) &= 0 & \text{in} & \Omega, \\ y\,(x,t) &= h_0\chi_O + (\omega_0 - v)\chi_\omega & \text{on} & \Gamma \times (0,T)\,, \end{cases} \tag{35}$$

satisfies

$$q(x,0,v) = 0 \tag{36}$$

Keywords: Linear parabolic equation, Sentinel method, Null controllabilty, Carleman inequality.

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Kirchhoff type equations with singular exponential nonlinearities

Mebarka Sattaf

Department of mathematics, Faculty of Exact Sciences, Djillali Liabes University, O. Box 89 Sidi Bel Abbes 22000, Algeria

E-mail: `sattafsattaf@gmail.com`

Abstract: In this work, We establish the existence of a positive solution for a nonlocal Kirchhoff problem of the type

$$\begin{cases} -M\left(\int\limits_{\Omega} |\nabla u|^N \, dx\right) \Delta_N u = \frac{f(x,u)}{|x|^\beta} & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{37}$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$ containing the origin $(N \geq 2)$,$0 \leq \beta < N$ and f, m are continuous functions that satisfy some assumptions.

Now, we are ready to state our main result.

Theorem 1. Under assumption $(M_1) - (M_3)$ and $(f_1) - (f_4)$, problem a positive solution $u \in W_0^{1,N}(\Omega)$.

We prove this Theorem by mountain pass Lemma and the Palais-Smale sequence.

Lemma 1. Assume that conditions (M_1) , $(f_1) - (f_3)$ hold, then there exist positive constants τ and ρ such that

$$E(u) \geq \tau > 0, \ \forall u \in W_0^{1,N}(\Omega) : \|u\| = \rho.$$

Proof:

From the assumptions, (f_1) ,(f_2) and (f_3) for $\varepsilon > 0$, there exists $C > 0$ and $q > N$ we have

$$\int_\Omega \frac{|F(x,u)|}{|x|^\gamma} dx \leq \varepsilon \int_\Omega \frac{|u|^N}{|x|^\gamma} dx + C \int_\Omega \frac{|u|^q e^{\alpha u^{N/N-1}}}{|x|^\gamma} dx \tag{38}$$

$$\leq \varepsilon C_1 \|u\|^N + C_2 \|u\|^q. \tag{39}$$

For $\|u\| = \rho$ where $R^{N/N-1} \leq \frac{\alpha_n}{2\alpha}(1 - \frac{\gamma}{N})$, thanks to Moser-Trudinger inequality and by (3) hence

$$E(u) \geq \left(\frac{m_0}{N} - C_1 \varepsilon\right) \|u\|^N - C_2 \|u\|^q,$$

Where $\tau = \left(\frac{m_0}{N} - C_1\varepsilon\right)\|u\|^N - C_2\|u\|^q$.

Lemma 2.
Assume that conditions (M_1) and (f_1) (f_4) hold. Then, there exists $e \in W_0^{1,N}(\Omega)$ with $\|e\| > \rho$ such that $I(e) < 0$.

Proof:
by (f_4), for $\eta > \max\{N, Nk + N\}$ there exist C_1, $C_2 > 0$ such that

$$F(x,t) \geq C_1 t^\eta - C_2 \; \forall (x,t) \in \Omega \times [0, +\infty). \tag{40}$$

from (M_1) and (4) we have

$$E(tu_0) \leq \begin{cases} \frac{b_0}{N} + \frac{b_1}{N}t^N + \frac{b_2}{Nk+N}t^{Nk+N} - C_1 t^\eta \int_\Omega \frac{{u_0}^\eta}{|x|^\beta}dx + C_2 \int_\Omega \frac{1}{|x|^\beta}dx & if k \neq -1 \\ \frac{b_0}{N} + \frac{b_1}{N}t + \frac{b_2}{N}\ln(t) - C_0 t^\eta \int_\Omega \frac{{u_0}^\eta}{|x|^\beta}dx + C_2 \int_\Omega \frac{1}{|x|^\beta}dx & if \; k = -1, \end{cases}$$

from which we conclude that $I(tu_0) \to -\infty$ as $t \to +\infty$, provided that $\eta > \max\{N, Nk + N\}$ Hence, the result follows by considering $e = t_* u_0$ for some $t_* > 0$ enough large.

Lemma 3.
Every Palais-Smale sequence of E is bounded in $W_0^{1,N}(\Omega)$

Keywords: Exponential critical growth; Kirchhoff equation; The mountain pass geometry ;Trudinger-Moser inequality .

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A Thermo-Electro-viscoelastic Contact Problem with Adhesion and Damage

Chougui Rachid [(1)], Lebri Nemira [(2)]

[(1)] Chougui Rachid, Department of Mathematics, Setif 1- University, 19000, Algeria
E-mail: chougui61@yahoo.fr

[(2)] Lebri Nemira, Applied Mathematics Laboratory, Department of Mathematics, Setif 1-University, 19000, Algeria
E-mail: nem_mat2000@yahoo.fr

Abstract: This talk is devoted to the study of the mathematical model involving a frictional contact between an electro-elasto-viscoplastic body with thermal effects and a conductive adhesive foundation. The process is mechanically dynamic and electrically static. The contact is modeled with a normal compliance where the adhesion is taken into account and a regularized electrical conductivity condition. We derive a variational formulation of the problem and prove its unique weak solution. The proof is based on nonlinear evolution equations with monotone operators, differential equations and fixed point arguments.

Keywords: piezoelectric materials, thermoviscoelastic, dynamic process, variational inequality, sub-differantial, fixed point.

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Non-linear memory of σ-evolution models with two dissipative terms

Mohamed KAINANE MEZADEK [(1),(2)]

[(1)] Mathematics Department, Faculty of Exact Sciences and Informatics, Hassiba Benbouali University, Chlef, P.B. 78, Ouled Fares, 02000, Chlef, Algeria.
E-mail: `med.kainane@univ-chlef.dz`

[(2)] Laboratory of Mathematics and Application (LMA), Hassiba Benbouali University, Chlef, P.B. 151, Hay Essalem, 02000, Chlef, Algeria.

Abstract: In this talk we study the global (in time) existence of small data solutions to the Cauchy problem for semilinear σ-evolution models with two dissipative terms, namely,

$$\begin{cases} u_{tt} + (-\Delta)^{\sigma} u + (-\Delta)^{\delta} u_t + (-\Delta)^{\theta} u_t = \int_0^t (t-\tau)^{-\gamma} f\big(u(\tau,\cdot)\big)\, d\tau, \\ u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), \end{cases} \tag{41}$$

where $\sigma \geq 1,\ 0 \leqslant \delta < \sigma/2 < \theta \leqslant \sigma,\ \gamma \in (0,1)$, and the data $(u_0, u_1) \in L^{m_1}(\mathbb{R}^n) \cap H^s_q(\mathbb{R}^n) \times L^{m_2}(\mathbb{R}^n) \cap H^{s-2\sigma}_q(\mathbb{R}^n),\ s \geq 0,\ , q \in [1,\infty),\ m_1,\ m_2 \in [1,q]$ and f is locally lipschitz function. We find the sharp critical exponent, under the assumption of small initial data, we show how the critical exponent is consequently modified for the problem. In particular, we obtain a new interplay between the fractional order of integration γ in the nonlinear memory term and the assumption that initial data are small in $(u_0, u_1) \in L^{m_1} \times L^{m_2}$, for some $m_1,\ m_2 \in [1,q]$.
The motivation of this talk is the influences of the parameter γ in the critical exponent to (1) and we are interested in connections between the critical exponent of power nonlinearity term and critical exponent of memory of nonlinearity term as $\gamma \to 1$, we prove the global (in time) existence of small data energy solutions to semilinear models (1). In the memory of nonlinearity we suppose $a \in [0, 2\sigma)$. So, we assume data u_1 from some energy space, but on the base of $q \in (1,\infty)$ and with additional regularity L^m, where $m \in [1,q)$. The memory of nonlinearity is allowed to be of classical type or of derivative type as well.

Keywords: Critical exponent, global in time existence, semi-linear evolution equations, structural damping, non-linear memory term.

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Elliptic problems with Robin boundary coeofficient - operator conditions in Hölder spaces: non commutative cases

Rabah Mohammed [(1)], Andasmas Maamar [(2)], Rabah Haoua [(3)], Ahmed Medeghri [(4)]

[(1)] Université Abdelhamid Ibn Badis, LMPA, 27000 Mostaganem, Algérie.
E-mail: `mohammed.rabah@univ-mosta.dz`

[(2)] Université Abdelhamid Ibn Badis, LMPA, 27000 Mostaganem, Algérie.
E-mail: `maamar.andasmas@univ-mosta.dz`

[(3)] Université Abdelhamid Ibn Badis, LMPA, 27000 Mostaganem, Algérie.
E-mail: `rabah.haoua@univ-mosta.dz`

[(4)] Université Abdelhamid Ibn Badis, LMPA, 27000 Mostaganem, Algérie.
E-mail: `ahmed.medeghri@univ-mosta.dz`

Abstract: we prove some new results on operational second order differential equations of elliptic type with general Robin boundary conditions in a non-commutative framework. The study is developed in Hölder spaces under some natural assumptions generalizing those in 1. We give necessary and sufficient conditions on the data to obtain a unique strict solution satisfying the maximal regularity property, see 3. This work completes the one given in 1, 2 and 3.

this talk is devoted to study the following general problem

$$\begin{cases} u''(x) + Au(x) - \omega u(x) = f(x), \quad x \in]0,1[\\ u'(0) - Hu(0) - \mu u(0) = d_0 \\ u(1) = u_1, \end{cases} \tag{42}$$

with $f \in C^{\theta}([0,1]; X)$, $0 < \theta < 1$, where X is a complex Banach space, d_0, u_1 are given elements in X and A is a closed linear operator of domain $D(A)$ are not necessarily dense in X. H is a closed linear operator in X, ω, μ are complex parameters.

Set:

$$A_{\omega} = A - \omega I \text{ and } H_{\mu} = H + \mu I.$$

We will seek for a strict solution u to (42), i.e. a function u such that:

$$u \in C^2([0,1]; X) \cap C([0,1]; D(A)) \text{ and } u(0) \in D(H_{\mu}).$$

The method is essentially based on Dunford calculus, interpolation spaces, the semigroup theory and some techniques as in 1 and 2.

Our main ellipticity assumption is the following: $[0,+\infty[\subset\rho(A)$ and $\exists\, C_A>0:$

$$\forall,\omega\geqslant 0,\quad \left\|(A-\omega I)^{-1}\right\|_{\mathcal{L}(X)}\leqslant\frac{C_A}{1+\omega}. \tag{43}$$

Here we do not assume the density of $D(A)$ in X. It is well known that $Q=-\sqrt{-A}$ and $Q_\omega=-\sqrt{-A+\omega I}$ are well defined and generate analytic semigroups $\left(e^{xQ}\right)_{x\geqslant 0}$, not necessarily strongly continuous in 0, see, for instance, C. Martinez 4. Note that, $\overline{D(Q)}=\overline{D(A)}$.

We get the following theorem as a result:
Theorem: Assume (43). Let $f\in C^\theta([0,1];X)$, with $0<\theta<1$ and $d_0,u_1\in X$. Then, for any $\omega\geqslant\omega_1^*$, we have

i) Problem (42) has a unique strict solution u if and only if

$$\begin{cases}(Q_\omega-H_\mu)^{-1}\left[d_0-Q_\omega^{-1}f(0)\right]\in D(Q^2)\\ Q_\omega^2(Q_\omega-H_\mu)^{-1}\left[d_0-Q_\omega^{-1}f(0)\right]+f(0)\in\overline{D(Q)}\\ u_1\in D(Q^2)\\ Q_\omega^2u_1+f(1)\in\overline{D(Q)}\end{cases}$$

ii) Problem (42) has a unique strict solution u satisfying the maximal regularity property u'', $A_\omega u\in C^\theta([0,1];X)$ if and only if

$$\begin{cases}(Q_\omega-H_\mu)^{-1}\left[d_0-Q_\omega^{-1}f(0)\right]\in D(Q^2)\\ Q_\omega^2(Q_\omega-H_\mu)^{-1}\left[d_0-Q_\omega^{-1}f(0)\right]+f(0)\in D_Q(\theta;+\infty)\\ u_1\in D(Q^2)\\ Q_\omega^2u_1+f(1)\in D_Q(\theta;+\infty)\end{cases}$$

Keywords: second-order elliptic differential equations; Robin boundary conditions; analytic semigroup.

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On the Stability of Singular Roesser State Space Models

Kamel BENYETTOU [(1)], Djillali BOUAGADA [(2)], Amine Mohammed GHEZZAR [(3)]

[1,2,3] Department of Mathematics and Computer Science
ACSY Team-Laboratory of Pure and Applied Mathematics
Abdelhamid Ibn Badis University Mostaganem
P.O.Box 227/118 University of Mostaganem, 27000 Mostaganem, Algeria
E-mail: `kamel.benyattou.etu@univ-mosta.dz` [1],
`djillali.bouagada@univ-mosta.dz` [2], `amine.ghezzar@univ-mosta.dz` [3]

Abstract: This work investigates new sufficient conditions for asymptotic stability for a two-dimensional singular continuous time systems described by Roesser models. The proposed approach is based on the characteristic polynomial and some linear matrix inequality $\mathcal{LMI}s$. Note that Roesser model is a two-dimensional systems which has two independents variables propagate the state in two independents direction. The obtained results are compared with existing work. Some illustrated examples and simulations have been established to show the applicability and accuracy of the proposed method.

The stability of the singular system is defined as follows

$$\lim_{t_1,t_2\to+\infty}\left\|\begin{pmatrix} x_h\left(t_1,t_2\right) \\ x_v\left(t_1,t_2\right)\end{pmatrix}\right\| = 0 \tag{44}$$

with the boundary condition $\sup_{t_2\mathbb{R}^+}\|x_h(0,t_2)\| < +\infty$ and $\sup_{t_1\mathbb{R}^+}\|x_v(t_1,0)\| < +\infty$.

Based on [1, 2] and [3], we give our main result as an approach to derive a new sufficient conditions for the asymptotic stability of the considered model.

Theorem . The singular $2D$ linear continuous time system is asymptotically stable if there exist a hermitian matrix X_0, X_1, X_2 with $X_0 \geq 0$, $X_1 \geq 0$, $X_2 \geq 0$ satisfying the following $\mathcal{LMI}s$:

$$(\tilde{A}-\tilde{E}diag(0,I_{n_2}))^T X_1\tilde{E}diag(I_{n_1},0) - \tilde{E}^T diag(I_{n_1},0)X_1(\tilde{A}-\tilde{E}diag(0,I_{n_2})) \succ 0 \tag{45}$$

$$\begin{bmatrix} \tilde{A}^T X_2\tilde{E}diag(0,I_{n_2}) - \tilde{E}^T diag(0,I_{n_2})X_2\tilde{A} & X_0 \\ \left(\tilde{E}^T diag(I_{n_1},0)X_2\tilde{E}diag(0,I_{n_2}) - \tilde{E}^T diag(0,I_{n_2})X_2\tilde{E}diag(I_{n_1},0)\right) & 0 \end{bmatrix} \succ 0 \tag{46}$$

Example Let us consider the singular Roesser system with $u(t_1,t_2) = 0$ and the system matrices

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 0.4165 & 1.5350 & -0.8697 & 0.8682 & -1.0616 \\ 0.0784 & -0.9935 & 0.8470 & -0.8316 & 1.0094 \\ 0.3563 & 6.8170 & -3.0333 & 4.8156 & -6.6059 \\ -0.6517 & 0.1915 & -0.1348 & 1.2781 & -1.2468 \\ 0.6636 & 2.1505 & -0.5888 & 1.7413 & -1.6655 \end{bmatrix} \tag{47}$$

By the use of our method we find that the $\mathcal{LMI}s$ in Theorem are feasible, and a feasible solution is as follows

$$X_0 = \begin{bmatrix} 1.3996 & 0.6940 & 0.6940 & 0.6940 & 0.6940 \\ 0.6940 & 1.3996 & 0.6940 & 0.6940 & 0.6940 \\ 0.6940 & 0.6940 & 1.3996 & 0.6940 & 0.6940 \\ 0.6940 & 0.6940 & 0.6940 & 1.3996 & 0.6940 \\ 0.6940 & 0.6940 & 0.6940 & 0.6940 & 1.3996 \end{bmatrix}, \tag{48}$$

$$X_1 = \begin{bmatrix} 0.0000 & 0.0000 & -0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & -0.0000 & -0.0000 \\ -0.0000 & 0.0000 & 0.9987 & 0.0131 & 0.0142 \\ 0.0000 & -0.0000 & 0.0131 & 0.9900 & 0.0067 \\ 0.0000 & -0.0000 & 0.0142 & 0.0067 & 1.0106 \end{bmatrix} \tag{49}$$

$$X_2 = \begin{bmatrix} 0.9991 & 0.0012 & 0.0019 & 0.0000 & -0.0071 \\ 0.0012 & 1.0000 & -0.0014 & -0.0000 & 0.0021 \\ 0.0019 & -0.0014 & 0.9963 & -0.0000 & 0.0106 \\ 0.0000 & -0.0000 & -0.00000.0000 & 0.0000 & \\ -0.0071 & 0.0021 & 0.0106 & 0.0000 & 0.9743 \end{bmatrix} \tag{50}$$

Keywords: Stability conditions, Singular systems, Linear matrix inequality.

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Singular nonlinear problems with natural growth in the gradient

Boussad Hamour [(1)]

[(1)] Laboratoire Equations aux Dérivées Partielles non Linéaires, Ecole Normale Supérieure, BP: 92 Vieux-Kouba, Alger
E-mail: `hamour@ens-kouba.dz`

Abstract: In this talk we consider the problem:

$$\begin{cases} -\mathrm{div}\,(a(x,u,Du) = H(x,u,Du) + \dfrac{a_0(x)}{|u|^\theta} + f(x) \quad \text{in } \Omega, \\ u = 0 \quad \text{on } \partial\Omega, \end{cases} \tag{51}$$

where Ω is an open bounded set of $\mathbb{R}^N$, $1 < p < N$. The function $a : \Omega \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}^N$ is a Carathéodory function which satisfies, for a.e. $x \in \Omega$, any $s \in \mathbb{R}$ and any $\xi, \xi' \in \mathbb{R}^N$, with $\xi \neq \xi'$:

$$\begin{cases} \big(a(x,s,\xi) - a(x,s,\xi')\big)(\xi - \xi') > 0, \\ a(x,s,\xi)\xi \geq \alpha|\xi|^p, \\ |a(x,s,\xi)| \leq \beta\big(b(x) + |s|^{p-2} + |\xi|^{p-1}\big), \end{cases}$$

for a given constant $\alpha > 0$, some constant $\beta > 0$, some nonnegative function $b \in L^{N/(p-1)}(\Omega)$. The function $a_0 \in L^{N/p}(\Omega)$, $a_0 > 0$, the source term $f \in L^{N/p}(\Omega)$ and the exponent $0 < \theta \leq 1$. Finally the function $H(x,s,\xi)$ is a Carathéodory function which satisfies:

$$-c_0\, a(x,s,\xi)\xi \leq H(x,s,\xi)\,\mathrm{sign}(s) \leq \gamma\, a(x,s,\xi)\xi \quad \text{a.e. } x \in \Omega, \forall s \in \mathbb{R}\ , \forall \xi \in \mathbb{R}^N.$$

For $\|a_0\|_q$ and $\|f\|_{N/p}$ sufficiently small, we prove the existence of at least one solution u of this problem which is moreover such that the function $\big(\exp(\gamma|u|) - 1\big)$ belongs to $W_0^{1,p}(\Omega)$.

A similar result has been proved in the quasilinear case where $p = 2$ and where the function $a(x,s,\xi)$ is assumed to have the form $a(x,s,\xi) = A(x)\xi$, with $A(x)$ is a matrix bounded entries and coercive.

In the present case the change of unknown function $v = \mu^{-1}\big(\exp(\mu|u|) - 1\big)\,\mathrm{sign}(u)$ transforms the equation (51) into a quasilinear equation with a quadratic term in Dv which satisfies a "sign condition, with the good sign", and a nonlinear term of zeroth-order "bad sign" which is superlinear and coercive.

The proof of our existence results follows the classical direct method of the calculus of variations. We use Schauder's theorem on an approximate problem to prove the existence

of a fixed point which is a solution of the approximate problem. In the other hand we obtain an a priori estimate of the solution of the approximate problem, which does not depend on the approximation.
We then pass to the limit thanks to the a priori estimate.

Keywords: Nonlinear problems, Singular terms, Natural growth in the gradient.

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Singular quasilinear elliptic systems with gradient dependence

Halima Dellouche
Abderrahmane Mira Bejaia University - ALGERIA
E-mail: `dellouche18@gmail.com`

Abstract: We study existence and regularity of positive solutions for a singular quasilinear elliptic system involving gradient term. The approach is based by comparison properties, a priori estimates and the Schauder's fixed point theorem.

Let $\Omega \subset \mathbb{R}^N$ $(N \geq 2)$ be a bounded domain with smooth boundary $\partial\Omega$. We deal with the following quasilinear elliptic system

$$\text{(P)} \qquad \begin{cases} -\Delta_{p_1} u = f_1(x, u, v, \nabla u, \nabla v) \text{ in } \Omega \\ -\Delta_{p_2} v = f_2(x, u, v, \nabla u, \nabla v) \text{ in } \Omega \\ u, v > 0 \quad \text{ in } \Omega \\ u, v = 0 \quad \text{ on } \partial\Omega. \end{cases}$$

The nonlinearity terms $f_i(x, u, v, \nabla u, \nabla v)$ can exhibit singularities when the variables u and v approach zero, and are subjected to the hypothesis:

(H_f) There exist constants $M_i, m_i > 0$, $\gamma_i, \theta_i \geq 0$, $r_i > N$ and $\alpha_i, \beta_i \in \mathbb{R}$ such that

$$m_i s_1^{\alpha_i} s_2^{\beta_i} \leq f_i(x, s_1, s_2, \xi_1, \xi_2) \leq M_i s_1^{\alpha_i} s_2^{\beta_i} + |\xi_1|^{\gamma_i} + |\xi_2|^{\theta_i}$$

for a.e. $x \in \Omega$, for all $s_1, s_2 > 0$, for all $\xi_1, \xi_2 \in \mathbb{R}^N$, with

$$|\alpha_i| + |\beta_i| \leqslant p_i - 1, \tag{52}$$

$$-1/r_i \leq \alpha_i + \beta_i < \frac{p_i - 1}{r_i} \quad \text{and} \quad \max\{\gamma_i, \theta_i\} < \frac{p_i - 1}{r_i}, \text{ for all } i = 1, 2. \tag{53}$$

The aim of this talk is to establish existence and regularity of (positive) solutions for quasilinear singular convective system (P) subjected to the growth condition (H_f). The main result is formulated as follows.

Theorem . Under assumption (H_f); system (P) admits a positive solution $(u, v) \in C_0^{1,\sigma}(\overline{\Omega}) \times C_0^{1,\sigma}(\overline{\Omega})$ for certain $\sigma \in (0, 1)$.

Corollary:

For $1 < p \leqslant N$ and $M > 0$; let $h : \Omega \times \mathbb{R} \times \mathbb{R}^N$ be a Caratheodory function satisfying

$$|h(x, s, \xi)| \leqslant M(d(x)^\mu + \xi^\gamma),$$

for a.e. $x \in \Omega$, for all $s \in \mathbb{R}$, and all $\xi \in \mathbb{R}^N$ with
$r > N$ and $\frac{-1}{r} < \mu < 0 \leqslant \gamma < \frac{p-1}{r}$
Then, there are constants $R > 0$ and $\sigma \in (0,1)$ such that all solutions $u \in W_0^{1,p}(\Omega)$ of Dirichlet problem

$$\begin{cases} -\Delta_{p_1} u = h(x, u, \nabla u) \text{ in } \Omega \\ u = 0 \quad \text{ on } \partial\Omega \end{cases}$$

belong to $C^{1,\sigma}(\overline{\Omega})$ and satisfy the estimate$\|u\|_{C^{1,\sigma}(\overline{\Omega})} < R$. Moreover, there is a constant $k_p > 0$; depending only on p and Ω ; such that

$$\|u\|_\infty \leqslant k_p \|h(x, u, \nabla u)\|_r^{\frac{1}{p-1}}.$$

Keywords: p-Laplacian; singular systems; regularity; convection terms; fixed point.
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Stabilization of a Coupled axially moving beam with a nonlinear tension

Billal Lekdim [(1),(2)], Ammar Khemmoudj [(2)]

[(1)] Department of Mathematics, University Ziane Achour of Djelfa, Djelfa 17000, Algeria
E-mail: `b.lekdim@univ-djelfa.dz`

[(2)] Laboratory of SDG, Faculty of Mathematics, University of Science and Technology Houari Boumediene, P.O. Box 32, El-Alia 16111, Bab Ezzouar, Algiers, Algeria
E-mail: `akhemmoudj@yahoo.fr`

Abstract: An axially moving beam in a two-dimensional space is considered with nonlinear tension. A suitable boundary control is applied at the free end of the beam to suppress the undesirable vibration. The exponential stability result is proven by Lyapunov method.

Keywords: axially moving beam, exponential stability, two-dimensional space, Lyapunov.

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Utilisation de l'opérateur de prolongement impair pour généraliser une suite des solutions approchées de l'équation de transport-diffusion dans $\mathbb{R}^d_+$

GHERDAOUI Rabah [(1)], TALEB Lynda [(2)],

[(1)] LMPA-University of Tizi-Ouzou
E-mail: `rabah.gherdaoui@ummto.dz`

[(2)] LMPA-University of Tizi-Ouzou
E-mail: `lytaleb@yahoo.fr`

Résumé: Comme il est bien connu que l'équation de transport-diffusion est une équation qui représente de nombreux phénomènes météorologiques (voir par exemple 1, 2,...), nous essayons donc de développer des résultats liés à cette équation. Pour cela, nous utilisons l'opérateur de prolongement impair pour construire une suite des solutions approchées de l'équation de transport-diffusion dans $\mathbb{R}^d_+$ à partir d'une suite des solutions approchées de l'équation de transport-diffusion dans $\mathbb{R}^d$ qui a été construite par le noyau de chaleur.

On considère l'équation de transport-diffusion

$$\partial_t u(t,x) + v(t,x)\cdot\nabla u(t,x) = \kappa\Delta u(t,x) + f(t,x,u(t,x)), \tag{54}$$

où $v(t,x)$ et $f(t,x,u)$ sont des fonctions données. Dans 3 une famille de solutions approchées pour (54) a été construite en utilisant le noyau de la chaleur $\Theta_n(.)$ (solution fondamentale de l'équation de la chaleur) sur chaque pas du temps discrétisé et leur convergence vers la solution du problème de Cauchy (dans $\mathbb{R}^d$) pour (54) avec la condition initiale a été démontrée. A partir de ce résultat, nous utilisons l'opérateur de prolongement impair Λ, afin de définir les solutions approchées $\widetilde{u}^{[n]}(t,x)$ de l'équation (54) avec des conditions initiale et aux limite dans $\mathbb{R}^d_+$. Nous considérons la discrétisation du temps

$$0 = t_0^{[n]} < t_1^{[n]} < \cdots < t_{k-1}^{[n]} < t_k^{[n]} < \cdots, \qquad t_k^{[n]} = k2^{-n} \equiv k\delta_n$$

et le noyau de la chaleur relatif à l'intervalle de temps δ_n et au coefficient de diffusion $\kappa > 0$

$$\Theta_n(x) = \frac{1}{(4\pi\delta_n\kappa)^{\frac{d}{2}}}\exp(-\frac{|x|^2}{4\delta_n\kappa}), \qquad x\in\mathbb{R}^d.$$

Nous introduisons aussi l'opérateur de prolongement impair

$$\Lambda(w(\cdot))(r) = \begin{cases} w(r), & \text{si } r>0 \\ 0, & \text{si } r=0 \\ -w(-r), & \text{si } r<0 \end{cases}.$$

Soit $u_0(x)$ une fonction donnée sur $\mathbb{R}^d_+$. On définit les solutions approchées $\widetilde{u}^{[n]}(t,x)$ par

$$u^{[n]}(t_0^{[n]},x) = u_0(x), \quad x \in \mathbb{R}^d_+,$$

$$u^{[n]}(t_k^{[n]},x) =$$

$$= \int_{\mathbb{R}^d} \Theta_n(y)\Lambda(u^{[n]}(t_{k-1}^{[n]}, x' - \delta_n v'(t_k^{[n]},x) - y', \cdot))(x_d - \delta_n v_d(t_k^{[n]},x) - y_d)dy'dy_d +$$

$$+\delta_n f(t_{k-1}^{[n]}, x, u^{[n]}(t_{k-1}^{[n]},x)), \qquad x \in \mathbb{R}^d_+, \quad k = 1, 2, \cdots,$$

$$u^{[n]}(t,x) = \frac{t_k^{[n]} - t}{\delta_n} u^{[n]}(t_{k-1}^{[n]},x) + \frac{t - t_{k-1}^{[n]}}{\delta_n} u^{[n]}(t_k^{[n]},x) \qquad \text{pour } \ t_{k-1}^{[n]} \le t \le t_k^{[n]}, \ \ x \in \mathbb{R}^d_+,$$

$$\widetilde{u}^{[n]}(t,x) = \int_{\mathbb{R}^d} \Theta_n(y)\Lambda(u^{[n]}(t, x' - y', \cdot))(x_d - y_d)dy.$$

Mots clés: Équation de transport-diffusion, approximation par un noyau de chaleur.

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Global existence of classical solutions to a class of Saint-Venant Equations

Riyadh Azib [(1)], Svetlin Georgiev [(2)], Arezki Kheloufi [(3)], Karima Mebarki [(4)]

[(1)] Department of Mathematics, Faculty of Exact Sciences, Laboratory of Applied Mathematics, Bejaia University 06000 Bejaia, Algeria.
E-mail : `azibriyadh@gmail.com`

[(2)] Department of Differential Equations, Faculty of Mathematics and Informatics, University of Sofia, Sofia, Bulgaria.
E-mail : `svetlingeorgiev1@gmail.com`

[(3)] Laboratory of Applied Mathematics, Bejaia University 06000 Bejaia, Algeria.
E-mail : `arezki.kheloufi@univ-bejaia.dz`

[(4)] Laboratory of Applied Mathematics, Faculty of Exact Sciences Bejaia University, 06000 Bejaia, Algeria.
E-mail : `karima.mebarki@univ-bejaia.dz`

Abstract : Nowadays, one of the most topics of active mathematical research is investigations of the existence of global classical solutions for non linear evolution equations. In this talk, we present a study of the Cauchy problem for a classical system of shallow water equations which describes long surface waves in a fluid of variable depth. This system was proposed in 1871 by Adhémar Jean-Claude Barré de Saint-Venant. Namely, we consider the following initial value problem for the Saint-Venant equations:

$$\begin{cases} \partial_t u + \partial_x(uv) & = & 0, \quad t \in (0,\infty), \quad x \in \mathbb{R}, \\ \partial_t(uv) + \partial_x\left(uv^2 + \frac{1}{2}ku^2\right) + ku\partial_x f(t,x) & = & 0, \quad t \in (0,\infty), \quad x \in \mathbb{R}, \\ u(0,x) & = & u_0(x), \quad x \in \mathbb{R}, \\ v(0,x) & = & v_0(x), \quad x \in \mathbb{R}, \end{cases} \tag{55}$$

where $k \in \mathbb{R}_+$ represents the gravitational constant, the initial conditions u_0, v_0 and the topography of the bottom f are given functions. Here the unknowns are $u = u(t,x)$ and $v = v(t,x)$, which denote respectively the depth and the average horizontal velocity of the fluid.

Here, we are especially interested in question of what conditions the initial data u_0, v_0 and the topography of the bottom f should be satisfy in order to ensure that Problem (55) has classical global solutions. By a classical solution to the Saint-Venant equations we mean a solution which is along with its derivatives that appear in the equations of class

$\mathcal{C}([0,\infty)\times\mathbb{R})$. In other words, (u,v) belongs to the space $\mathcal{C}^1([0,\infty)\times\mathbb{R})\times\mathcal{C}^1([0,\infty)\times\mathbb{R})$ of continuously differentiable functions on $[0,\infty)\times\mathbb{R}$.

The main assumptions on the functions u_0, v_0, and f are the following :

(H1) $u_0, v_0 \in \mathcal{C}^1(\mathbb{R})$, $0 \leq u_0, v_0 \leq B$ on $\mathbb{R}$ for some positive constant B.

(H2) $f \in \mathcal{C}([0,\infty), \mathcal{C}^1(\mathbb{R}))$, $0 \leq |\partial_x f| \leq B$ on $[0,\infty)\times\mathbb{R}$.

Our approach is based on the use of the fixed point theory for the sum of operators in Banach spaces. Hereafter, the main steps to obtain our results. First, we will present a new topological approach which uses fixed point abstract theory of the sum of two operators. Then, we give some properties of solutions of Problem (55). These properties will be used to prove our main results concerning existence and multiplicity of solutions for the Saint-Venant system (55). Finally, an example illustrating our main results will be given.

Keywords : Saint-Venant equations, classical solution, fixed point, initial value problem.

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Existence, uniqueness and reiterated homogenization of quasilinear hyperbolic-parabolic problems in perforated domains

DEHAMNIA Abdelhakim, HADDADOU Hamid
Laboratory of physics mathematics and applications, (ENS), Algiers, Algeria.
E-mail: abdelhakimdehamnia@gmail.com
LCSI laboratory, Ecole nationale suprieure d'Informatique (ESI ex INI), Algiers, Algeria.
E-mail: h_haddadou@esi.dz

Abstract: The main purpose of the present talk is to study the asymptotic behavior (when $\varepsilon \to 0$) of the solution related to a quasilinear hyperbolic-parabolic problem given in a periodically perforated domain with two spatial and one temporal scales. Under certain assumptions on the problem's coefficients and based on a priori estimates and compactness results, we establish homogenization results by using the multiscale convergence method. We study here, the homogenization of the quasilinear hyperbolic-parabolic equations

$$\begin{cases} \alpha^\varepsilon(x)u_\varepsilon'' + \beta^\varepsilon(x,t)u_\varepsilon' - \Delta u_\varepsilon' - div\left(a\left(\frac{x}{\varepsilon_1}, \frac{x}{\varepsilon_2}, \frac{t}{\varepsilon_1'}, \nabla u_\varepsilon\right)\right) = f(x,t) & \text{in } \Omega_\varepsilon \times (0,T), \\ a\left(\frac{x}{\varepsilon_1}, \frac{x}{\varepsilon_2}, \frac{t}{\varepsilon_1'}, \nabla u_\varepsilon\right).\nu = 0,\ \partial_\nu u_\varepsilon = 0 & \text{on } \partial S_\varepsilon \times (0,T), \\ u_\varepsilon = 0 & \text{on } \partial\Omega \times (0,T), \\ u_\varepsilon(x,0) = h_\varepsilon(x),\ \alpha^\varepsilon(x)u_\varepsilon'(x,0) = \sqrt{\alpha^\varepsilon(x)}k_\varepsilon(x) & \text{for } x \in \Omega_\varepsilon. \end{cases} \tag{56}$$

where Ω_ε is a periodically perforated domain and

$$\alpha^\varepsilon(x) = \alpha(\frac{x}{\varepsilon_1}, \frac{x}{\varepsilon_2}),\ \beta^\varepsilon(x,t) = \beta(\frac{x}{\varepsilon_1}, \frac{x}{\varepsilon_2}, \frac{t}{\varepsilon_1'}) \text{ and } a\left(\frac{x}{\varepsilon_1}, \frac{x}{\varepsilon_2}, \frac{t}{\varepsilon_1'}, \xi\right) = \sigma(\frac{x}{\varepsilon_1}, \frac{x}{\varepsilon_2}, \frac{t}{\varepsilon_1'})|\xi|^{\rho-2}\xi.$$

$\alpha, \beta, \sigma, h_\varepsilon$ and k_ε are functions which satisfy certain hypothesis. Here, Ω is a bounded open subset of a space $\mathbb{R}^N (N \geq 2)$, T is a real positive number and $2 \leq \rho < \infty$. The set Ω_ε is a domain perforated on two scales defined for example in figure 1. The main results of this work are as follow:

Theorem 1. For any fixed real number ε and under certain assumptions on the problem's coefficients, there exist a unique solution $u \in L^\infty(0,T;V_\varepsilon^\rho)$ of the problem (56) and a constant C such that

$$\|\sqrt{\alpha_\varepsilon}u_\varepsilon'\|_{\Omega_\varepsilon}^2 + \int_0^T ||u_\varepsilon'||_{\Omega_\varepsilon}^2 dt + \|u_\varepsilon'\|_{V_\varepsilon}^2 + \|u_\varepsilon\|_{V_\varepsilon^\rho}^2 \leq C\left(||f||_{(0,T)\times\Omega_\varepsilon}^2 + ||k_\varepsilon||_{\Omega_\varepsilon}^2 + ||\nabla h_\varepsilon||_{\Omega_\varepsilon}^2\right).$$

Theorem 2. Let $(u_\varepsilon)_{\varepsilon>0}$ be the sequence of solution to (56). Under certain assumptions on the problem's coefficients, there exist three functions $(u_0, u_1, u_2) \in \mathcal{V}^\rho$ such that, as

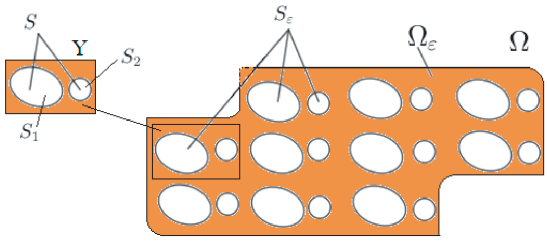

Figure 1: The domain Ω_ε in the two-dimensional case

$\varepsilon \to 0$, $\mathcal{P}^\varepsilon u_\varepsilon$ converges weakly to u_0 in $L^\rho(0,T;W_0^{1,\rho}(\Omega))$ and the triplet (u_0,u_1,u_2) is the unique solution of

$$(\widehat{\alpha}u_0'',v_0)_{L^2(\Omega_T)} + (\widehat{\beta}u_0',v_0)_{L^2(\Omega_T)} + \frac{1}{|Y^*\times Z^*|}\int\limits_{\Omega_T}\int\limits_{G}\left(\frac{\partial}{\partial t}\boldsymbol{Du}\right)(\boldsymbol{Dv})\,d\gamma dxdt$$

$$+\frac{1}{|Y^*\times Z^*|}\int\limits_{\Omega_T}\int\limits_{G}\tilde{b}\,(\boldsymbol{Du})\,(\boldsymbol{Dv})\,d\gamma dxdt = (f,v_0)_{L^2(\Omega_T)}, \quad \text{for all } v=(v_0,v_1,v_2)\in\mathcal{V}^\rho,$$

where $\tilde{b}(\xi) = \left(\int\limits_G \widehat{\sigma}d\gamma\right)|\xi|^{\rho-2}\xi$, $\boldsymbol{Du} = \nabla u_0 + \nabla_y u_1 + \nabla_z u_2$, $\boldsymbol{Dv} = \nabla v_0 + \nabla_y v_1 + \nabla_z v_2$, $G = Y\times Z\times\mathcal{T}$ and $\mathcal{V}^\rho = V^\rho\times L^\rho(\Omega\times(0,T);W_\#^{1,\rho}(Y^*))\times L^\rho(\Omega\times(0,T);L^\rho(Y\times\mathcal{T};W_\#^{1,\rho}(Z^*)))$.

Keywords: Quasilinear hyperbolic-parabolic equations, Homogenization, Multiscale convergence method.

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Fractional Differential Equations

New Duffing Fractional Differential Oscillator of Sequential Type

ABDELNEBI Amira [(1)]

[(1)] Laboratory of Pure and Applied Mathematics,
Faculty of SEI, University of Mostaganem
B.P. 227, 27000, Mostaganem, Algeria
E-mail: `Amira.math27@gmail.com`

Abstract: In this talk, by introducing the fractional derivative in the sense of Caputo-Hadamard and Hadamard integral operator, we study a nonlinear sequential fractional problem of Duffing oscillator type. The contraction mapping principle and Schaefer fixed point theorem are applied to prove the existence and the uniqueness of solutions, then Ulam-Hyers and generalized Ulam-Hyers stabilities are analyzed. At the end, an illustrative example is presented.

The present presentation deals with a more complicated problem of nonlinear fractional differential of Duffing type. By injecting the derivatives of Caputo-Hadamard not only on the left side of the equation, but on right hand side of the problem too, this consideration makes the considered problem more interesting, this is in one hand. On the other hand, the motivation of the present paper can be seen also in the fact that Caputo-Hadamard approach has many advantages with respect to the usual Hadamard approach. So let us consider the following problem:

$$\begin{cases} D^{\beta}(D^{\alpha}x(t)) + k\ f(t, D^{\alpha}x(t)) + g(t, x(t), D^{p}x(t)) = h(t). \\ x(0) = A^{*} \in \mathbb{R}, \qquad D^{\alpha}x(0) = B^{*} \in \mathbb{R}, \qquad x(e) = C^{*} \in \mathbb{R}, \\ 0 < p < \alpha < 1, \quad 1 < \beta < 2, t \in I, \end{cases}$$

where $D^{\alpha}, D^{\beta}, D^{p}$ are the Caputo-Hadamard fractional derivatives, $I = [0, e]$, k is a real constant, the functions f, g and h are continuous.

More precisely, we will recall some preliminary related to fractional calculus concepts for our problem, and by proving two main theorems, we apply the fixed point theory to study the questions of existence and uniqueness of solutions for the considered problem. Then, other results around stability in the sense of Ulam-Hyers and generalized Ulam-Hyers stability will be analyzed. At the end, we will present an example to validate the theoretical results.

Keywords: Duffing differential equation, Fixed point, Uniqueness, Ulam-Hyers stability.

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Boundary Value Problems for Fractional q-Difference Equations

Nadia Allouch [(1)]

[(1)] Laboratory of Pure and Applied Mathematics,
Faculty of SEI, University of Mostaganem
B.P. 227, 27000, Mostaganem, Algeria
E-mail : `nadia.allouch.etu@univ-mosta.dz`

This work is in collaboration with **Prof. S. Hamani** and **Prof. J. Henderson**.

Abstract : In this paper, we studied the existence and uniqueness of solutions for a class of boundary value problems for fractional equations involving the Caputo fractional q-difference derivative of order $0 < \alpha \leq 1$. Ours results are given by applying some standard fixed point theorems.
This communication deals with the existence of solutions for the boundary value problem for fractional q-difference equations of the form

$$({}^{C}D_q^{\alpha} y)(t) = f(t, y(t)), \text{ for a.e. } t \in J = [0, T], \quad 0 < \alpha \leq 1, \tag{57}$$

$$ay(0) + by(T) = c, \tag{58}$$

where $T > 0$, $q \in (0, 1)$, ${}^{C}D_q^{\alpha}$ is the Caputo fractional q-difference derivative of order $0 < \alpha \leq 1$, $f : [0, T] \times \mathbb{R} \to \mathbb{R}$ is a given function and a, b and c are real constants such that $a + b \neq 0$.
We give two existence results, one based on the Banach fixed point theorem, another based on Schaefer's fixed point theorem. Finally, we present an example.
Keywords : Fractional q-difference equations; Caputo fractional q-difference derivative; existence; fixed point.

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Stability and Hopf bifurcation of glycolysis model involving Caputo fractional derivative

Naziha Belmahi [(1)], Nabil Shawagfeh [(2)]

[(1)] University of sciences and technologiy Mohammed Boudiaf, Oran, Algeria
E-mail: NazZiha-ID@hotmail.fr

[(2)] University of Jordan, Amman, Jordan
E-mail: shawagnt@ju.edu.jo

Abstract: It has been observed that most of the biological models have memory or what we call after effects, such effects in the systems are often neglected. For this reason, the researchers considered the fractional derivative to play a significant role in recognizing and understanding these effects on the real life models dynamics. It should be noted that the fractional order dynamical systems are gaining popularity due to its various applications. As it is well known, many mathematical, engineering, biological, chemical, physical models can be described more accurately by fractional derivatives than traditional order derivatives. In this talk, we introduce a new model of glycolysis phenomenon involving the Caputo time fractional derivative, which is a generalization of the classical model given by Selkov in 1968.

The Selkov model has attracted the interest of many researchers, since it is a simple description for glycolysis which is a complex process by which energy is extracted from sugar. It has been observed that under suitable circumstances the rate at which products of glycolysis accumulate shows oscillations in time although the input rate of sugar to the system is constant.

We start by deriving sufficient conditions for the local asymptotic stability of the equilibrium point of the proposed system using the theorem of 'Matignon'. Then, we pass to the analysis of the Hopf bifurcation existence that permits us to prove the existence of a limit cycle, hence the oscillating behaviour of Selkov fractional model. Finally, We present some numerical simulations to show the advantages of using a fractional order derivative on the dynamics of the system and to validate our main findings.

The expansion of the stability region is what distinguishes the generalization of the problem to the fractional, this has been shown clearly in this presentation, and we validated it with numerical simulations.

Keywords: Fractional derivative; Selkov model; Stability; Hopf bifurcation; limit cycle.

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Implicit fractional integro-differential equations involving ψ-Caputo fractional derivative

Ichrak Bouacida [(1)], Mourad Kerboua [(2)], Sami Segni[(3)]

[(1)] LMAM, University 8 Mai 1945, Guelma.
E-mail: `ichrakbouacida@gmail.com`

[(2)] LMAM, University 8 Mai 1945, Guelma.
E-mail: `kerbouamourad@gmail.com`

[(3)] LMAM, University 8 Mai 1945, Guelma.
E-mail: `segnianis@gmail.com`

Abstract: In this talk, we study the existence and uniqueness for a class of implicit integro-differential equation involving ψ-Caputo fractional derivative of the form

$$\begin{cases} {}^{C}D_{0^+}^{\alpha,\psi}x(t) &= f\left(t, x(t), {}^{C}D_{0^+}^{\alpha,\psi}x(t), I_{0^+}^{\alpha,\psi}x(t)\right), t \in J \\ x(0) &= x_0, \end{cases} \tag{1}$$

where ${}^{C}D_{0^+}^{\alpha,\psi}$ is ψ-Caputo fractional derivative of order $\frac{1}{2} < \alpha < 1$, and $I_{0^+}^{\alpha,\psi}$ is the ψ-Riemann-Liouville fractional integrals of orders α; $f : J \times \mathbb{R}^3 \to \mathbb{R}$ is a given continuous function, and $J = [0, T]$, $T > 0$. The state $x(.)$ take value in Banach space X.

(H1) There exist constants $L_1, L_3 > 0$, and $0 < L_2 < 1$ such that

$$|f(t, x_1, y_1, z_1) - f(t, x_2, y_2, z_2)| \leq L_1|x_1 - x_2| + L_2|y_1 - y_2| + L_3|z_1 - z_2|,$$

for any $x_i, y_i, z_i \in \mathbb{R}$, $i = 1, 2$ and $t \in J$.

(H2) is a consequence of (H1).

(H2) There exist non negative continuous functions $h_1, h_2, h_3, h_4 \in \mathbb{R}$ such that

$$|f(t, x, y, z)| \leq h_1(t) + h_2|x| + h_3|y| + h_4|z|. \quad x, y, z \in \mathbb{R},\ t \in J,$$

with $h_1^* = \underset{t\in J}{sup} h_1(t)$, $h_2^* = \underset{t\in J}{sup} h_2(t)$, $h_3^* = \underset{t\in J}{sup} h_3(t)$, $h_4^* = \underset{t\in J}{sup} h_4(t)$.

(H3) $f(t, x, y, z) \leq q(t) = f(t, 0, 0, 0)$, $(t, x, y, z) \in J \times \mathbb{R}^3$ where $q(t) \in C(J, \mathbb{R}^+)$.

For computational convenience, we use the following notations:

$$\Psi(v, \lambda) = \frac{(\psi(\lambda) - \psi(0))^{\gamma+v-1}}{\Gamma(\gamma + v)}$$

$$\Lambda_1(v,\lambda) = \Gamma(v)\Big[L_1\Psi(v,\lambda) + L_2KK'\Psi(v-1,\lambda) + L_3K\Psi(v-1,\lambda)\Big]$$

Theorem 1 *Let $f : J \times R^3 \to R$ be a continuous function satisfying (H1). If*

$$\Theta_1 = 1 + (\psi(T) - \psi(0))^{1-\gamma}\Lambda_1(\alpha, T) < 1,$$

then problem (1) has a unique solution $x \in C_{1-\gamma,\psi}$ on J.

Theorem 2 *Let $f : J \times R^3 \to R$ be a continuous function satisfying (H2). Then problem (1) has at least one solution on J.*

Keywords: ψ-Caputo fractional derivative; Fractional evolution equation; Existence; Uniqueness.

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Existence and uniqueness of solutions of a terminal value problem for fractional-order differential equations with advanced arguments

Mohammed Derhab
Dynamic Systems and Applications Laboratory
Department of Mathematics
Faculty of Sciences
University Abou-Bekr Belkaid Tlemcen
B.P.119, Tlemcen
13000, Algeria
E-mail: derhab@yahoo.fr

Abstract: In this talk, we study of the existence and uniqueness of solutions for the following terminal value problem

$$\begin{cases} {}^{C}D_{-}^{\alpha}u(x) = f(x, u(x), u(\theta(x))), \ x \in [1, +\infty), \\ \lim\limits_{x\to+\infty} u(x) = a, \end{cases} \tag{59}$$

where ${}^{C}D_{-}^{\alpha}$ is the modified fractional Caputo derivative of order α with $0 < \alpha < 1$, $f : [1, +\infty) \times \mathbb{R}^2 \to \mathbb{R}$ and $\theta : [1, +\infty) \to [1, +\infty)$ are continuous and $a \in \mathbb{R}$.

Definition 1. *([2]). Let $g : [1, +\infty) \to \mathbb{R}$ be a function. For $\alpha > 0$ the modified Liouville fractional Integral of order α of g is defined by*

$$J_{-}^{\alpha}g(x) = \frac{1}{\Gamma(\alpha)} \int_{x}^{+\infty} (xt)^{1-\alpha} \frac{g(t)}{t^2 (t-x)^{1-\alpha}} dt, \text{ for all } x \in [1, +\infty).$$

Definition 2. *([2]). Let $g : [1, +\infty) \to \mathbb{R}$ be a function. For $0 < \alpha < 1$ the modified Liouville fractional derivative of order α of g is defined by*

$$\begin{aligned} D_{-}^{\alpha}g(x) &= -x^2 \frac{d}{dx} J_{-}^{1-\alpha}g(x) \\ &= -\tfrac{x^2}{\Gamma(1-\alpha)} \frac{d}{dx} \int_{x}^{+\infty} (xt)^{\alpha} \frac{g(t)}{t^2 (t-x)^{\alpha}} dt, \text{ for all } x \in [1, +\infty). \end{aligned}$$

We note $C(1; +\infty)$ the following space

$$C(1; +\infty) = \left\{ u \in C([1, +\infty), \mathbb{R}), \ \lim_{x\to+\infty} u(x) = a \right\}.$$

Definition 3. *Let $g : [1, +\infty) \to \mathbb{R}$ be a function such that $g \in C(1; +\infty)$ with $\lim_{x \to +\infty} g(x) = b$. For $0 < \alpha < 1$ the modified Caputo fractional derivative of order α of g is defined by*

$$\begin{aligned} {}^{C}D_{-}^{\alpha} g(x) &= -J_{-}^{1-\alpha}(b - g(x)) \\ &= -\tfrac{x^2}{\Gamma(1-\alpha)} \frac{d}{dx} \int_{x}^{+\infty} (xt)^{\alpha} \frac{(b - g(t))}{t^2 (t - x)^{\alpha}} dt, \text{ for all } x \in [1, +\infty). \end{aligned}$$

Also, we have the following definition

Definition 4. *Let $g : [1, +\infty) \to \mathbb{R}$ be a function. For $0 < \alpha < 1$ the modified Caputo fractional derivative of order α of g is defined by*

$$\begin{aligned} {}^{C}D_{-}^{\alpha} g(x) &= -J_{-}^{1-\alpha}(x^2 g'(x)) \\ &= -\tfrac{1}{\Gamma(1-\alpha)} \int_{x}^{+\infty} (xt)^{\alpha} \frac{g'(t)}{(t - x)^{\alpha}} dt, \text{ for all } x \in [1, +\infty). \end{aligned}$$

Definition 5. *We say that u is a solution for the problem **(59)** if $u \in C(1; +\infty)$, ${}^{C}D_{-}^{\alpha} u \in C([1, +\infty), \mathbb{R})$ and u satisfies **(59)**.*

Theorem 1. *Assume that the following hypothesis is satisfied*
There exist $L_1 > 0$ and $L_2 > 0$ such that

$$|f(x, u_1, v_1) - f(x, u_2, v_2)| \leq L_1 |u_1 - u_2| + L_2 |v_1 - v_2|,$$

for all $x \in [1, +\infty)$ and u_i, $v_i \in \mathbb{R}$ for $i = 1, 2$.
Then the terminal value problem (59) admits a unique solution.

Keywords: Modified fractional Liouville integral; modified fractional Caputo derivative; terminal value problem; advanced arguments; Banach's fixed point theorem.

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Positives solutions for a fractional differential equation with nonlocal integro-differential boundary conditions on unbounded interval

Abdellatif GHENDIR AOUN

Department of Mathematics, Faculty of Exact Sciences,
Hamma Lakhdar University, 39000 El-Oued, Algeria
E-mail: ghendirmaths@gmail.com

Abstract: The objective of this topic is to present the existence of positive solutions for a fractional order boundary value problem of the fractional differential equation with a homogeneous fractional integral boundary condition at the left end and a non-local fractional integral boundary condition at the right end.It is a subject of fractional differential equations which is one of the topics of the RDOPDE 22 conference. This problem is as follows:

$$\begin{cases} D_{0^+}^{\alpha} u(t) + \varphi(t) f(t, u(t)) = 0, & t > 0, \\ I_{0^+}^{2-\alpha} u(0) = 0, & \lim\limits_{t \to +\infty} D_{0^+}^{\alpha-1} u(t) = \gamma I_{0^+}^{\beta} u(\eta), \end{cases} \tag{60}$$

where $1 < \alpha \leq 2$ and $\beta,\ \gamma,\ \eta > 0$. The function $f : [0, +\infty) \times [0, +\infty) \to [0, +\infty)$ is continuous and $\varphi : [0, +\infty) \to [0, +\infty)$ is continuous, not identically zero on any closed subinterval of $[0, +\infty)$, and $\varphi \in L^1[0, +\infty)$.

First, we study the corresponding Green's function associated with problem (60) and describe some of its properties. Second, we demonstrate some technical lemmas that will be needed later. Finally we discussed our main result of existence of positive solutions. It is based on the fixed point theorem of Avery-Henderson [1].
The following conditions for obtaining positive solutions have been determined:

$(H1)$ $0 < \gamma\eta^{\alpha+\beta-1} < \Gamma(\alpha + \beta)$.

$(H2)$ $f(t, (1 + t^{\alpha-1})u) < ma$, pour $t \in [0, +\infty)$, $u \leq a$ avec

$$m = \frac{\Gamma(\alpha)(\Gamma(\alpha + \beta) - \gamma\eta^{\alpha+\beta-1})}{\Gamma(\alpha + \beta) \int_0^{+\infty} \varphi(s) ds}.$$

$(H3)$ $f(t, (1 + t^{\alpha-1})u) > \frac{b}{m'}$, pour $t \in [\frac{1}{k}, k]$, $b \leq u \leq Mb$ avec

$$m' = \frac{\lambda(k) \int_{\frac{1}{k}}^{k} \varphi(s) ds}{\Gamma(\alpha) k^{\alpha-1}}.$$

$(H4)$ $f(t,(1+t^{\alpha-1})u) \leq mr$, pour $t \in [0,+\infty)$, $u \leq r$.

$(H5)$ $f(t,(1+t^{\alpha-1})u) > \frac{r}{m'}$, pour $t \in [\frac{1}{k},\, k]$, $r \leq u \leq Mr$.

$(H6)$ $f(t,(1+t^{\alpha-1})u) < bm$, pour $t \in [0,\, +\infty)$, $b \leq u \leq Mb$.

$(H7)$ $f(t,(1+t^{\alpha-1})u) > \frac{a}{m'}$, pour $t \in [\frac{1}{k},\, k]$, $\frac{a}{M} \leq u \leq a$.

Keywords: Boundary value problem; fractional differential equation; positive solution; infinite interval; nonlocal conditions; fixed point theorem.

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Existence, uniqueness and monotonicity of positive solutions for hybrid fractional integro-differential equations

Moussa HAOUES [(1)], Abdelouhab ARDJOUNI [(2)]

[(1)] Souk Ahras University, Laboratory of Informatics and Mathematics P.O. Box 1553, Souk Ahras, 41000, Algeria.
E-mail: moussa.haoues@yahoo.com

[(2)] Department of Mathematics and Informatics, Souk Ahras University P.O. Box 1553, Souk Ahras, 41000, Algeria.
E-mail: abd-ardjouni@yahoo.fr

Abstract: In this talk, we study the existence, uniqueness and monotonicity of positive solutions for hybrid nonlinear fractional integro-differential equations by the method of upper and lower solutions and using Dhage and Banach fixed point theorems.

Fractional differential equations have been of great interest recently. It is caused both by the intensive development of the theory of fractional calculus itself and by the applications. Particularly, the existence of positive solution of fractional differential equations is considered in depth in the last years. Although the tools of fractional calculus have been available and applicable to various fields of study (science, engineering,physics, chemistry, biology, medicine, atomic...). Hybrid differential equations arise from a variety of different areas of applied mathematics and physics, e.g., in the detection of a curved beam having a constant or varying cross section, a three-layer beam, electromagnetic waves or gravity driven flows and so on. Dhage and Lakshmikantham in [1] discussed the existence of solutions for the following first order hybrid differential equation

$$\begin{cases} \dfrac{d}{dt}\left(\dfrac{x(t)}{g(t,x(t))}\right) = f(t,x(t)) \text{ a.e. } t \in [t_0, t_0+T], \\ x(t_0) = x_0 \in \mathbb{R}. \end{cases}$$

where $t_0, T \in \mathbb{R}$ with $T > 0$, $g : [t_0, t_0+T] \times \mathbb{R} \to \mathbb{R} \setminus \{0\}$ and $f : [t_0, t_0+T] \times \mathbb{R} \to \mathbb{R}$ are continuous functions. By using the fixed point theorem in Banach algebra, the authors obtained the existence results.

Let $\mathfrak{J} = [0, a]$ be a closed and bounded interval of the real line $\mathbb{R}$ for some $a \in \mathbb{R}$ with $a > 0$. The hybrid fractional differential equations

$$\begin{cases} D^{\alpha}\left(\dfrac{x(t)}{g(t,x(t))}\right) = f(t,x(t)) \text{ a.e. } t \in \mathfrak{J}, \\ x(0) = 0. \end{cases}$$

has been investigated in [4] where D^{α} is the Riemann-Liouville fractional derivative of order $0 < \alpha < 1$, $f : \mathfrak{J} \times \mathbb{R} \to \mathbb{R}$, and $g : \mathfrak{J} \times \mathbb{R} \to \mathbb{R} \diagdown \{0\}$, are continuous functions. By employing the fixed point theorem in Banach algebra the authors obtained the existence of a solution.

Let $J = [t_0, T]$. Matar[2] investigated the existence, uniqueness and monotonicity of positive solutions for the following hybrid fractional differential equation

$$\begin{cases} {}^{C}D_{t_0}^{\alpha}\left(\dfrac{x(t)}{g(t, x(t))}\right) = f(t, x(t)), \ t \in J, \\ x(t_0) = \theta \geq 0, \end{cases}$$

where ${}^{C}D_{t_0}^{\alpha}$ is the Caputo fractional derivative of order $0 < \alpha \leq 1$, $f : J \times \mathbb{R} \to \mathbb{R}$ and $g : J \times \mathbb{R} \to \mathbb{R} \diagdown \{0\}$ are given continuous functions such that $g(t_0, x(t_0)) = \lambda > 0$. By using the method of the upper and lower solution and the Dhage and Banach fixed point theorems, the authors obtained the existence, uniqueness and monotonicity of a positive solution. Inspired and motivated by the works mentioned above and some recent studies on hybrid fractional differential equations, we consider the existense, uniqueness and monotonicity of positive solutions for the following hybrid nonlinear fractional integro-differential equation

$$\begin{cases} {}^{C}D_{t_0}^{\alpha}\left(\dfrac{x(t)}{p(t) + \frac{1}{\Gamma(\beta)}\int_{t_0}^{t}(t-s)^{\beta-1} g(s, x(s))\, ds}\right) = f(t, x(t)), \ t \in J, \\ x(t_0) = p(t_0)\,\theta \geq 0, \end{cases} \tag{61}$$

where $0 < \alpha, \beta \leq 1$, $f, g : J \times \mathbb{R} \to \mathbb{R}$,and $p : J \to \mathbb{R}$,are given continuous functions. To show the existence, uniqueness and monotonicity of positive solution, we transform (61) into an integral equation and then by the method of upper and lower solutions and use Dhage and Banach fixed point theorems.

Keywords: Fractional integro-differential equation, Fixed point theorems, Existence and uniqueness, Positivity, Monotonicity.

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Uniqueness and Mittag-Leffler-Ulam-stability results for sequential Riemann-Liouville-Caputo fractional Duffing problem

Houas mohamed

Laboratory FIMA, UDBKM, Khemis Miliana university, Algeria

E-mail: houas_mohamed@yahoo.fr

Abstract: Differential equations with fractional derivative operators have attracted great attention in the last years, these fractional differential equations arise in the modelling of various problems in sciences and engineering [4]. Recently, several authors have studied the existence, uniqueness and different types of Mittag-Leffler-Ulam-stability of solutions for differential equations of fractional order [1,3]. In recent years, many scholars have exposed attention in the field of theory of nonlinear fractional differential equations. One of the very important nonlinear differential equations is the Duffing equation [2]. In this present talk, we discuss the existence, uniqueness and Mittag-Leffler-Ulam-stability of solutions for sequential Riemann-Liouville-Caputo fractional Duffing equation

$$\begin{cases} {}_{R.L}D^{\alpha}\left[{}_{C}D^{\beta}\left[{}_{C}D^{\gamma}s\left(t\right)\right]\right] = k\left(t\right) - Mg\left(t, s\left(t\right), {}_{C}D^{\delta}s\left(t\right)\right) - h\left(t, s\left(t\right), I^{\eta}s\left(t\right)\right), \\ s\left(0\right) = 0,\ {}_{C}D^{\beta}\left[{}_{C}D^{\gamma}s\left(1\right)\right] = 0,\ s\left(1\right) = A,\ A \in \mathbb{R}, \\ t \in J := [0,1],\ 0 < \alpha, \beta, \gamma \leq 1,\ \delta < \gamma,\ M > 0, \eta > 0, \end{cases} \tag{62}$$

where ${}_{R.L}D^{\alpha}, {}_{C}D^{\mu}, \mu \in \{\beta, \gamma\}$, denote the Riemann-Liouville and Caputo fractional derivatives, $g, h : J \times \mathbb{R}^2 \to \mathbb{R}$ and $k : J \to \mathbb{R}$ are given continuous functions. The operator ${}_{R.L}D^{\alpha}$ is the fractional derivative in the sense of Riemann-Liouville, defined by ${}_{R.L}D^{\alpha}s\left(t\right) = \frac{d^n}{dt^n}\left(I^{n-\alpha}\left[s\left(t\right)\right]\right)$, $n = [\alpha] + 1$. The operator ${}_{C}D^{\alpha}$ is the fractional derivative in the sense of Caputo, defined by ${}_{C}D^{\mu}s\left(t\right) = I^{n-\alpha}\left[\frac{d^n}{dt^n}\left(s\left(t\right)\right)\right]$ and the Riemann-Liouville fractional integral of order $\alpha > 0$, defined by $I^{\eta}s\left(t\right) = \frac{1}{\Gamma(\eta)}\int_0^t \left(t-r\right)^{\eta-1} s\left(r\right) dr$, $t > 0$.

Let $W = \left\{s : s \in C\left(J, \mathbb{R}\right) \text{ and } {}_{C}D^{\delta}s \in C\left(J, \mathbb{R}\right)\right\}$ denotes the space equipped with the norm $\|s\|_W = \|s\| + \left\|{}_{C}D^{\delta}s\right\|$, where $\|s\| = \sup_{t\in J}\left|s\left(t\right)\right|$ and $\left\|{}_{C}D^{\delta}s\right\| = \sup_{t\in J}\left|{}_{C}D^{\delta}s\left(t\right)\right|$. It is clear that $(W, \|.\|_W)$ is a Banach space.

Lemma 2. *Lemma: Suppose that $m \in C\left([0,1], \mathbb{R}\right)$. Then, the fractional problem*

$$\begin{cases} {}_{R.L}D^{\alpha}\left[{}_{C}D^{\beta}\left[{}_{C}D^{\gamma}s\left(t\right)\right]\right] = m\left(t\right), 0 < \alpha, \beta, \gamma < 1,\ t \in J, \\ s\left(0\right) = 0,\ {}_{C}D^{\beta}\left[{}_{C}D^{\gamma}s\left(1\right)\right] = 0,\ s\left(1\right) = A,\ A \in \mathbb{R}, \end{cases}$$

admits the following solution:

$$\begin{aligned} s(t) &= \tfrac{1}{\Gamma(\alpha+\beta+\gamma)} \int_0^t (t-r)^{\alpha+\beta+\gamma-1} m(r)\, dr + \tfrac{t^{\gamma}-t^{\alpha+\beta+\gamma-1}}{\Gamma(\alpha+\beta+\gamma)} \int_0^1 (1-r)^{\alpha-1} m(r)\, dr \\ &\quad - \tfrac{t^{\gamma}}{\Gamma(\alpha+\beta+\gamma)} \int_0^1 (1-r)^{\alpha+\beta+\gamma-1} m(r)\, dr + t^{\gamma} B. \end{aligned}$$

By using Banach's fixed point theorem, we will establish find a unique solution of the fractional Duffing problem (1).

Theorem 3. *Theorem 1: Let $g, h : J \times \mathbb{R}^2 \to \mathbb{R}$ and $k : J \to \mathbb{R}$ be continuous functions. In addition we assume that:*

(H_1) : There exist a constant $\omega > 0, \varpi > 0$ such that for all $t \in J$ and $s_l, z_l \in \mathbb{R}, l = 1, 2,$

$$\begin{aligned} |\varphi(t, s_1, s_2) - \varphi(t, z_1, z_2)| &\leq \omega_1 (|s_1 - z_1| + |s_2 - z_2|), \\ |\psi(t, s_1, s_2) - \psi(t, z_1, z_2)| &\leq \varpi (|s_1 - z_1| + |s_2 - z_2|). \end{aligned}$$

If $M\omega + \varpi \left(1 + \frac{1}{\Gamma(\eta+1)}\right) < \frac{1}{\nabla + \nabla^*}$, where $\nabla := \frac{2}{\Gamma(\alpha+\beta+\gamma+1)} + \frac{2}{\alpha\Gamma(\alpha+\beta+\gamma)}$ and $\nabla^* := \frac{1}{\Gamma(\alpha+\beta+\gamma-\delta+1)} + \frac{1}{\alpha\Gamma(\alpha+\beta+\gamma-\delta)} + \frac{\Gamma(\gamma+1)}{\Gamma(\gamma-\delta+1)} \left(\frac{1}{\alpha\Gamma(\alpha+\beta+\gamma)} + \frac{1}{\Gamma(\alpha+\beta+\gamma+1)}\right)$. Then the fractional Duffing problem (1) has a unique solution on J.

Now, we present the Mittag-Leffler-Ulam-Hyers stability for the fractional Duffing problem (1).

Definition 6. *Definition: The fractional Duffing problem (1) is Mittag-Leffler-Ulam-Hyers stable, with respect to $E_{\alpha+\beta+\gamma}$ if there exists a real number π such that for each $\rho > 0$ and for each solution $y \in W$ of the inequality $\left|{}_{R.L}D^{\alpha}\left[{}_{C}D^{\beta}\left[{}_{C}D^{\gamma} s(t)\right]\right] - m(t)\right| \leq \rho, t \in J$, there exists a solution $s \in W$ of the problem (1) with $\|y(t) - s(t)\|_W \leq \pi\rho E_{\alpha+\beta+\gamma}[t]$, $t \in J$.*

Theorem 4. *Theorem 2: Assume that $g, h : J \times \mathbb{R}^2 \to \mathbb{R}, k : J \to \mathbb{R}$ are continuous functions and suppose that (H_1) holds. Then the fractional Duffing problem (1) is Mittag-Leffler-Ulam-Hyers stable.*

Keywords: Fractional derivative, fixed point, existence, Duffing equation, Mittag-Leffler-Ulam stability.

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Study of positive periodic solutions for nonlinear neutral differential equation using Krasnoselskii's - Burton fixed point

Chahra Kechar [(1)], Abdelouaheb Ardjouni [(2)],

[(1)] Department of Mathematics and Informatics Souk Ahras, Algeria
E-mail: chahra95kechar@gmail.com

[(2)] Department of Mathematics, Faculty of Sciences, University of Annaba, Algeria
E-mail: abd-ardjouni@yahoo.fr

Abstract:

In this talk, we use a modification of Krasnoselskii's fixed point theorem introduced by Burton (see [6] Theorem3) to establish new results on the existence and positivity of solutions for the totally nonlinear neutral periodic differential equation. We invert the equation to construct a sum of a completely continuous map and a large contraction which is suitable for the application of a modification of Krasnoselskii's theorem.

Keywords:

fixed point theory, large contraction, totally nonlinear neutral differential equation, periodic.

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Global mild solution for fractional integro-differential equations with state-dependent nonlocal conditions

Sara Litimein [(1)], Zohra Bouteffal [(2)]

[(1)] Djillali Liabes University of Sidi Bel-Abbès, P.O. Box 89, Sidi Bel-Abbès 22000, Algeria.
E-mail: `sara_litimein@yahoo.fr`

[(2)] Department of Mathematics, Faculty of Exact Sciences, Mustapha Stambouli University of Mascara, P. O. Box 305, 29000, Algeria.
E-mail: `zohra.boutefal@univ-mascara.dz`

Abstract: In this talk we deal with the existence of mild solutions for non-linear fractional integro-differential equations with state-dependent nonlocal condition of the form

$$y'(t) - \int_0^t \frac{(t-s)^{\alpha-2}}{\Gamma(\alpha-1)} Ay(s)ds = f(t, y_{\rho(t,y_t)}), \quad \text{a.e. } t \in \mathbb{R}_+ := [0, +\infty), \tag{63}$$

$$y_0 = G(\sigma(y), y) \in \mathcal{C} = C([-r, 0], E), \tag{64}$$

where $1 < \alpha < 2$ and $A : D(A) \subset E \to E$ is a closed linear operator, and $(E, \|\cdot\|)$ is a Banach space.$f : \mathbb{R}_+ \times \mathcal{C} \to E$, $\sigma : C([-r, +\infty), E) \to \mathbb{R}_+$, $G : \mathbb{R}_+ \times C([-r, +\infty), E) \to E$ and $\rho : \mathbb{R}_+ \times \mathcal{C} \to \mathbb{R}$, are suitable functions. For any continuous function y defined on $[-r, +\infty)$ and any $t \in [0, +\infty)$, the technique used is a generalization of the classical Darbo fixed point theorem for Fréchet spaces associated with the concept of measures of noncompactness.
To prove our main result we need the following hypotheses

$(H1)$ There exists a constant $M > 1$ such that

$$\|S(t)\|_{B(E)} \leq M \text{ for every } t \in \mathbb{R}_+.$$

$(H2)$ The function $t \longmapsto f(t, y)$ is measurable on $\mathbb{R}_+$ for each $y \in E$, and the function $y \longmapsto f(t, y)$ is continuous on E for a.e $t \in \mathbb{R}_+$

$(H3)$ There exists a function $p \in L^1(\mathbb{R}_+; \mathbb{R}_+)$ and a continuous nondecreasing function $\psi : \mathbb{R}_+ \to [0, \infty)$ such that

$$\|f(t, y)\| \leq p(t)\ \psi(\|y\|) \text{ for a.e. } t \in \mathbb{R}_+ \text{ and each } y \in \mathcal{C},$$

$(H4)$ For each bounded and measurable set $B \subset E$ and for each $t \in \mathbb{R}_+$, we have

$$\mu(f(t, B)) \leq p(t)\mu(B).$$

$(H5)$ There exists $L > 0$ such that

$$\|G(\sigma(y), y)\| \leq L(1 + \|y\|) \text{ for each } y \in \mathcal{C}$$

$(H6)$ There exists $K > 0$ such that

$$\mu(G(\sigma(y)), B) \leq K\mu(B),$$

where μ is a measure of noncompactness on the Banach space E,

$(H7)$ For each $n \in \mathbb{N}$, there exists R_n a positive real number such that

$$ML(1 + R_n) + nM\psi(R_n)p_n^* \leq R_n.$$

For $n \in \mathbb{N}$, let

$$p_n^* = \sup_{t \in [0,n]} p(t)$$

and define on $C(\mathbb{R}_+)$ the family of measure of noncompactness by

$$\mu_n(D) = \sup_{t \in [0,n]} \mu(D(t));$$

and $D(t) = \{v(t) \in E; v \in D\}; t \in [0, n]$

Theorem 5. *Assume* $(H1) - (H7)$ *are satisfied, and*

$$\frac{4Mp_n^*}{1 - 2MK} < 1$$

for each $n \in \mathbb{N}$. *Then the problem* $(66) - (64)$ *has at least one mild solution.*

Keywords: Integro-differential Equations, solution operator, mild solution, fixed point, state-dependent nonlocal condition, measure of noncompactness.

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On Caputo fractional differential equation with an evolution problem by the subdifferential operator

Soumia Saïdi [(1)]

[(1)] LMPA Laboratory, Department of Mathematics, Mohammed Seddik Ben Yahia University, Jijel, Algeria
E-mail: soumiasaidi44@gmail.com

Abstract: In recent years, fractional differential equations and differential inclusions have proved to be crucial tools in modeling many physical and economical phenomena. Actually, there has been a significant development in fractional differential theory and its applications. In the case of systems involving the Caputo fractional derivative, an important piece of literature can be found.

In the current contribution, we are interested in a system coupled by a differential inclusion governed by the time-dependent subdifferential operator and a fractional differential equation (in a real Hilbert space H) formulated by

$$-\dot{u}(t) \in \partial\varphi(t, u(t)) + f(t, x(t)) + F(t, u(t)) \text{ a.e. } t \in [0, T]$$

$$^cD^\alpha x(t) = u(t), \quad t \in [0, T]$$

$$u(0) = u_0 \in dom\varphi(0, \cdot)$$

$$x(0) = x_0 \in H,$$

where $^cD^\alpha x$ denotes the Caputo fractional derivative of order $\alpha > 0$ of the function x. The real-valued map $\varphi(t, \cdot)$ from H into $[0, +\infty]$ is proper, lower semi-continuous, convex, and satisfies an assumption expressed in term of its conjugate function $\varphi^*(t, \cdot)$.
We denote by $\partial\varphi(t, \cdot)$ the subdifferential of $\varphi(t, \cdot)$ and by $dom\varphi(t, \cdot)$ its domain, for each $t \in [0, T]$. The single-valued perturbation $f : [0, T] \times H \to H$ is a Carathéodory map satisfying a suitable growth condition, while the set-valued map $F : [0, T] \times H \rightrightarrows H$ takes non-empty convex weakly compact values.

In our development, we combine the existence result for differential inclusions involving time-dependent subdifferential operators, an important lemma in fractional differential theory, and the fixed point theorem. From our main result, we deduce the related study for systems coupled by the sweeping process.

There remain several coupled systems by the evolution problems governed by subdifferential operators with fractional differential equations to be investigated in the line of recent works cited in references.

Keywords: Coupled system, differential inclusion, subdifferential operator, Caputo fractional derivative.

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Existence and uniqueness results for SICA model with fractional order

Nedjoua Zine [(1)], Bayour Benaoumeur [(2)], Delfim F.M. Torres [(3)]

[(1)] Department of mathematics , University of Mascara, Algeria
E-mail: `nadjoua.zine@univ-mascara.dz`

[(2)] Department of mathematics , University of Mascara, Algeria
E-mail: `b.bayour@univ-mascara.dz`

[(3)] Department of mathematics , University of Aveiro, Portugal
E-mail: `delfim@ua.pt`

Abstract: The purpose of this talk is to present existence and uniqueness for the epidemic model SICA (Susceptible-Infectious- Chronic-AIDS) for HIV/AIDS transmission dynamic with varying population size in a homogeneously mixing population, given by a system of four ordinary differential equations. We consider SICA models given by system with fractional order derivative in Caputo sense. We concern the required results by applying some basic theorems to prove the existence of the solution via fixed point theory and further to examine the uniqueness of the model variables.

In this work, we consider a SICA (Susceptible-Infectious-Chronic-AIDS) mathematical model for transmission dynamic of the human immunodeficiency virus HIV/AIDS with varying population size in a homogeneously mixing population which spreading continuously all over the world and there have been few generator which continue it, given by a system of four differential equations. Different scholars have treated this model in different cases.

In the present work, we have to study the following model with non-integer order of derivative with $0 < \alpha \leq 1$ which is given by

$$\begin{cases} {}_{t_0}^{C}D_t^{\alpha} w(t) = \Lambda - \beta(x(t) + \eta_y y(t) + \eta_z z(t))w(t) - \mu w(t), \\ {}_{t_0}^{C}D_t^{\alpha} x(t) = \beta(x(t) + \eta_y y(t) + \eta_z z(t))w(t) - \xi_3 x(t) + \omega y(t) + \gamma z(t), \\ {}_{t_0}^{C}D_t^{\alpha} y(t) = \gamma x(t) - \xi_2 y(t), \\ {}_{t_0}^{C}D_t^{\alpha} z(t) = \gamma x(t) - \xi_1 z(t) \end{cases}$$

where Λ, β, η_y, η_z, μ, ω, γ, ξ_1, ξ_2, ξ_3 are non-negative and the total population $N(t) = w(t) + x(t) + y(t) + z(t)$ at time $t \geq 0$. The corresponding derivative is taken in Caputo sense. Furthermore for the corresponding results, existence theory and uniqueness of solu-

tion are supplied by using fixed point theory.

With the help of Banach and Schauder theorems, we have investigated HIV/AIDS model with fractional derivatives.

Keywords: SICA model;HIV/AIDS ;Fractional derivative; existence and uniqueness; fixed point theory .

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Mathematical Physics and Modeling in Physics

Spherical caps conjecture in Minkowski space

Mohammed Abdelmalek [(1)],

[(1)] School of Management of Tlemcen-Algeria
E-mail: `abdelmalekmhd@yahoo.fr`

Abstract:

Given M^{n+1} an oriented Riemannian manifold, and M^n a compact oriented hypersurface embedded in M^{n+1}. Denoting by $x_1, ..., x_n$ its principal curvatures.

For $1 \leq k \leq n-1$, we define the higher order mean curvature H_k of M^n, as :

$$\binom{n}{k} H_k = \sum_{i_1 \prec \ldots \prec i_k} x_{i_1} \ldots x_{i_k}, \qquad 1 \leq k \leq n$$

For instance, $H_1 = H$ and H_n are respectively the mean and Gauss curvatures of the hypersurface.

The Alexandrov's spherical caps conjecture [1] states that the only compact hyersurface embedded in $\mathbb{R}^{n+1}$ with constant higher order mean curvature H_r and spherical boundary are the hyperplanar disc (if $H_r = 0$) and the spherical caps (if $H_r \neq 0$) .

In [3], the authors generalized the above result for hypersurfaces embedded in an oriented Riemannian manifold.

Notice that this theorem is not true if the hypersurfaces is immersed and not embedded.

In this talk, we give a partial answer of the Alexandrov spherical caps conjecture for hypersurfaces embedded in the Minkowski space. We prove that under sufficient conditions a compact oriented hypersurface with constant higher order mean curvature and spherical boundary embedded in the Minkonski space must be a part of the pseudo sphere.

For the prove we use the moving plane method (Alexandrov reflexion method).

We give in the end some examples and open problems.

Keywords: Newton transformations, Higher order mean curvature, Pseudo Riemanian manifolds.

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General decay for a viscoelastic translational Timoshenko system

Amirouche Berkani[(1)]

[(1)] Faculty of sciences and applied sciences, University Akli Mohand Oulhadj of Bouira, 10000 Bouira, Algeria.
E-mail: aberkanid@gmail.com

Abstract: In this talk, we study the existence and asymptotic behaviour of solutions for a a cantilevered Timoshenko beam. The beam is viscoelastic and subject to a translational displacement. The problem can be modeled by a set of partial differential equations (PDEs) taking into account therefore the dynamic boundary condition. Based on the standard Faedo-Galerkin method, we prove the well-posedness of our problem, then, under a suitable boundary control, we prove an arbitrary decay of the energy of the system for a large class of relaxation functions using the multiplier method.

The aim of this talk is to study the uniform stability of a Timoshenko beam with memory.

One end of the beam is fixed to a base in a translational motion while a tip mass is attached at its free end. The dynamic of the problem can be described by the following system

$$\begin{cases} m\xi_{tt}(t) + \rho_1 \displaystyle\int_0^L \big(\xi_{tt}(t) + w_{tt}(x,t)\big)dx + m_E\big(\xi_{tt}(t) + w_{tt}(L,t)\big) = \Phi(t), \quad t \in \mathbb{R}^+, \\ \rho_1\big(\xi_{tt}(t) + w_{tt}(x,t)\big) - k\big(w_{xx}(x,t) + \Theta_x(x,t)\big) = 0, \\ \rho_2\Theta_{tt}(x,t) - b\Theta_{xx}(x,t) + \displaystyle\int_0^t q(t-\tau)\Theta_{xx}(\tau)d\tau + k\big(w_x(x,t) + \Theta(x,t)\big) = 0, \end{cases}$$

for all $0 \leq x \leq L$, $t \in \mathbb{R}^+$, with the boundary conditions

$$\begin{cases} w(0,t) = \Theta(0,t) = 0, \quad t \in \mathbb{R}^+, \\ -k\big(w_x(L,t) + \Theta(L,t)\big) = m_E\big(w_{tt}(L,t) + \xi_{tt}(t)\big), \quad t \in \mathbb{R}^+, \\ b\Theta_x(L,t) - \displaystyle\int_0^t q(t-\tau)\Theta_x(L,\tau)d\tau = -J\Theta_{tt}(L,t), \quad t \in \mathbb{R}^+ \end{cases}$$

and the initial data

$$\begin{cases} \xi(0) = \xi^0,\ \xi_t(0) = \xi^1,\ \ w(x,0) = w^0(x),\ \ \Theta(x,0) = \Theta^0(x), \\ w_t(x,0) = w^1(x),\ \ \Theta_t(x,0) = \Theta^1(x),\ \ 0 \leq x \leq L, \end{cases}$$

where t denotes the times variable and x is the space variable along the beam of length L, ξ is the displacement of the translational base, w is the beam transversal displacement and Θ

is the rotational angle of the beam. The constants ρ_1, ρ_2, b and k are the mass density, the moment mass inertia, the rigidity coefficient (of the cross-section) and the shear modulus of elasticity, respectively. The coefficients m and m_E denotes respectively, the mass of the translational base and the mass attached at the free end of the beam with rotational J.

The convolution term in the third Eq. of first system represents the viscoelastic damping or the dependence on the history and the kernel q involved there is the relaxation function.

Now, our result reads as follows.

Theorem 6. *Under the control force $\Phi(t)$ applied at the base motion. Then, there exist positive constants Λ and ϱ such that*

$$E(t) \leq \Lambda \psi^{-\varrho}(t), \quad t \geq 0$$

if $\lim_{t \longrightarrow +\infty} \delta(t) = 0$ *and*

$$E(t) \leq \Lambda e^{-\varrho t}, \quad t \geq 0$$

if $\lim_{t \longrightarrow +\infty} \delta(t) \neq 0$.

Keywords: Timoshenko beam; well-posedness; Faedo-Galerkin method; general decay; relaxation function; viscoelasticity.

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Mathematical model of the obesity impact on COVID-19 severity

Boubekeur Maroua Amel [(1)] and Belhamiti Omar [(1)]
[(1)] University of Mostaganem Algeria
E-mail: `maroua.boubekeur.etu@univ-mosta.dz`
E-mail: `Omar.belhamiti@univ-mosta.dz`

Abstract: The Covid-19 epidemic was alerted by WHO in December 2019, and was declared a public health emergency of international concern (USPPI) by the same organization on January 30, 2020. As of October 17, 2021, more than 241 million cases had been confirmed and nearly 5 million deaths worldwide.
The global spread is very rapid, with 170 countries now reporting at least one case. It is very important to understand the dynamics of the epidemic's transmission early in order to better control its evolution and assess the effectiveness of control measures .

Many studies have established that several factors have a surprising correlation with higher mortality in individuals with Covid-19: arterial hypertension and smoking, obesity, diabetes, cardiac and pulmonary pathology. Over the past two years, many mathematical modeling studies of covid-19 associated with other chronic diseases have emerged, among these works.

In this talk, we propose a mathematical model that highlight the very negative effect of the COVID-19 pandemic on overweight and obese people.

This model takes into account different disease states and is represented mathematically by a nonlinear temporal system of ordinary differential equations It is divided into ten compartmental classes, namely susceptible non-obese class $S(t)$, exposed non-obese class $E(t)$, asymptomatic COVID-19 infected class $I_a(t)$, symptomatic COVID-19 infected class $I_s(t)$, the hospitalized infected class $I_h(t)$, obese class $O(t)$, exposed obese class $E_O(t)$, the class of obese was hospitalized with a COVID-19 infection $I_{Oh}(t)$, recovered class $R(t)$ and death class $D(t)$.

It was found that the proposed model has two equilibrium points; the disease-free equilibrium point (DFE) and the endemic equilibrium point (E_1). Stability analysis of the equilibrium points shows (E_0) is locally asymptotically stable whenever the basic reproduction number,$R_0 < 1$ and (E_1) is locally asymptotically stable whenever $R_0 > 1$. Numerical simulations are presented to explain the usefulness of the proposed model and confirm numerically the theoretical results gained above.

Keywords: **Obesity**, **Covid-19**, **Model Validation**, **Stability Analysis**.

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A frictional contact problem with wear and damage for elastic viscoplastic materials

Chouia Abdallah [(1)],Azeb Ahmed Abdelaziz [(2)]

[(1)] Mathematics Department, Hamma Lakhdar University of El Oued, Algeria
E-mail: `chouia-abdallah@univ-eloued.dz`

[(2)] Mathematics Department, Hamma Lakhdar University of El Oued, Algeria
E-mail: `aziz-azebahmed@univ-eloued.dz`

Abstract: We consider a quasistatic contact problem for an elastic-viscoplastic body with wear and damage between a elastic-viscoplastic body and a rigid obstacle. The contact is frictional and bilateral which results in the wear and damage of contacting surface. The evolution of the wear function is described with Archard's law.The evolution of the damage is described by an inclusion of parabolic type. We establish a variational formulation for the model and we prove the existence of a unique weak solution to the problem. The proof is based on a classical existence and uniqueness result on parabolic inequalities, differential equations and fixed point argument.

Keywords: quasistatic process, elastic viscoplastic materials, bilateral contact, friction and damage, existence and uniqueness, fixed point and weak solution

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Mathematical analysis of a dynamic piezoelectric contact problem with damage and adhesion

KHEZZANI RIMI .
Department of Mathematics, Operators Theory and PDE Foundations and Applications Laboratory University of El Oued, P.O. Box 789, El Oued 39000, Algeria
E-mail: khezzani-rimi@univ-eloued.dz

Abstract: In this talk, we study a mathematical problem for dynamic contact between two piezoelectric bodies with normal compliance, adhesion and damage. The damage of the material caused by elastic deformations. The evolution of the damage is described by an inclusion of parabolic type. The evolution of the bonding field is described by a first order differential equation. We derive variational formulation for the model and prove an existence and uniqueness result of the weak solution. The proof is based on arguments of time dependent variational inequalities, parabolic inequalities and Banach fixed point theorem.
Keywords: Dynamic process, thermo-piezoelectric materials, normal compliance, fixed point, damage field, adhesion field.

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Existence results for non-instantaneous impulsive stochastic integro-differential equations with nonlocal conditions

Melati Oussama [(1)], Slama Abdeldjalil [(2)], Abdelghani Ouahab [(3)]

[(1)] Laboratory of Mathematics Modeling and Applications, University of Adrar, National Road No. 06, Adrar 01000, Algeria
E-mail: `ous.melati@univ-adrar.edu.dz`

[(2)] Laboratory of Mathematics Modeling and Applications, University of Adrar, National Road No. 06, Adrar 01000, Algeria
E-mail: `slama_dj@yahoo.fr`

[(3)] Laboratory of Mathematics. University of Sidi-Bel-Abbes
E-mail: `agh_ouahab@yahoo.fr`

Abstract: In this talk we discuss the existence of mild solutions of non-instantaneous stochastic impulsive integro-differential equations with nonlocal conditions. The results are obtained by using Kuratowskii measure of noncompactness, resolvent operator and a generalized Darbo's fixed point theorem. An example is also given to illustrate the obtained results.

Keywords: Measure of noncompactness; stochastic integro-differential equations; non-instantaneous impulses; resolvent operator; fixed point theory.

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On the class of binormal operators

Safa Menkad [(1)], Sohir Zid [(2)]

[(1)] LTM, Department of Mathematics, University of Batna 2, Algeria
E-mail: `s.menkad@univ-batna2.dz`

[(2)] LTM, Department of Mathematics, University of Batna 2, Algeria
E-mail: `s.zid@univ-batna2.dz`

Abstract: Let $\mathcal{H}$ be a complex Hilbert space and $\mathcal{B}(\mathcal{H})$ the algebra of all bounded linear operators on H. An operator $T \in \mathcal{B}(\mathcal{H})$ is said to be normal if $TT^* = T^*T$, quasinormal if T commutes with T^*T and binormal if TT^* and T^*T commute. the class of binormal operators was introduced by Campbell in 1972. It is easy to see that normal $\Longrightarrow$ quasinormal $\Longrightarrow$ binormal and the inverse implications do not hold. It is well known that for every operator $T \in \mathcal{B}(\mathcal{H})$, there is a unique factorization $T = U|T|$, where $\mathcal{N}(U) = \mathcal{N}(T) = \mathcal{N}(|T|)$, U is a partial isometry, i.e. $UU^*U = U$ and $|T| = (T^*T)^{\frac{1}{2}}$ is the modulus of T. This factorization is called the polar decomposition of T. Related to this decomposition, the Aluthge transform of $T \in \mathcal{B}(\mathcal{H})$ is defined as

$$\Delta(T) = |T|^{\frac{1}{2}} U |T|^{\frac{1}{2}}.$$

This transform was introduced in [1] by Aluthge, during the investigating on the properties of p-hyponormal operators. More generally, for $\lambda \in [0, 1]$, Okubo defined the λ-Aluthge transform of T in [5], by

$$\Delta_\lambda(T) = |T|^\lambda U |T|^{1-\lambda}.$$

Notice that $\Delta_0(T) = T$ and $\Delta_1(T) = |T|U$ is known as is known as Duggal's transform. These transforms have been studied in many different contexts and considered by a number of authors. One of the interests of the Aluthge transform lies in the fact that it respects many properties of the original operator. Throughout the remainder of this paper, we denote by $\delta(\mathcal{H})$ the class of operator $T \in \mathcal{B}(\mathcal{H})$ which satisfies $U^2|T| = |T|U^2$. Clearly, quasinormal operators belong to $\delta(\mathcal{H})$ but the converse is not true in genaral. In this paper, firstly, we provide a condition under which an operator in $\delta(\mathcal{H})$ becomes quasinormal. Secondly, we show that an invertible operator T belongs to the class $\delta(\mathcal{H})$ if and only if $\Delta_1(T^{-1}) = (\Delta_1(T))^{-1}$. Afterwards, we give examples and discuss how this class of operators is distinct from the class of binormal operators. Finally, We prove that, if T is invertible in $\delta(\mathcal{H})$, then T is binormal if and only if $\Delta_\lambda(T^{-1}) = (\Delta_\lambda(T))^{-1}$, for $\lambda \in]0, 1[$.

Also, in this paper, we will investigate on some relations between the class of binormal operators and other usual classes of operators via λ-Aluthge Transform .

Keywords: Hilbert space , Binormal operator, Invertible operator, quasinormal, λ-Aluthge Transform .

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The behavior of spinless particles within the fractional dimensional space

Hadjer Merad

Laboratory of Mathematics, Informatics and Systems (LAMIS)
Larbi Tebessi University - Tebessa, Algeria
E-mail: meradhad@gmail.com

Abstract: The aim of this talk is to obtain an exact solution of spinless particles with relativistic energy subjected to the action of a scalar potential and a vector potential within the fractional- dimensional space, where the momentum and position operators satisfies the Heisenberg algebras of R-deformed model. Therefore, a number of problems have been solved, and in every instance, the wavefunction are expressed in terms of the special functions, and the corresponding energy spectrum are exactly given and drawn up in light of the deformation parameters, hence explains the confinement in law dimension.
In essence the presentation is structured as follows: Firstly, we present a review on the R-deformed Heisenberg algebra for the fractional-dimensional used in the calculations. Secondly, we expose the exact solution of The Klein Gordon equation with mixed scalar and vector linear in the fractional-dimensional space.
Keywords: Fractional-dimensional space, Bessel differential operator, R-deformed Heisenberg algebras, special functions.
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Exponential stability of neural networks with time delays and non-Lipschitz activations

Sakina Othmani [(1)], Nasser-eddine Tatar [(2)]

[(1)] Laboratory of SDG, Faculty of Mathematics University of Science and Technology Houari Boumediene P.O. Box 32, El-Alia 16111, Bab Ezzouar, Algeria
E-mail: `othmani.sakinaa@gmail.com`

[(2)] Department of Mathematics and Statistics, King Fahd University of Petroleum and Minerals, Interdisciplinary Research Center for Intelligent Manufacturing & Robotics, Dhahran 31261, Saudi Arabia
E-mail: `tatarn@kfupm.edu.sa`

Abstract: Neural networks are a product of Artificial Intelligence, which have a variety of applications in several fields such as biology, economy, finance, and many engineering fields. As an extension of the well-known Hopfield neural networks, Kosko in 1987 introduced a new class of recurrent networks called bidirectional associative memory (BAM) neural networks. Their design encompasses two interconnected hidden neuronal layers in which neurons in a single layer do not connect. These networks are widely applied in various areas such as pattern recognition, signal and image processing, automatic control, associative memory, and so forth. In hardware implementation, delays appear due to the finite switching speed of the amplifiers as the neurons communicate together. These may cause oscillations, divergences, and instabilities, which are detrimental effects on systems. Neural networks commonly have a spatial span due to the multiplicity of parallel paths with different size and length axons, and thus a propagation delay distribution occurs along a time interval. The retarded BAM neural network model we are interested in is described by the following differential equations:

$$\begin{cases} x_i'(t) = -c_i(x_i(t)) + \sum\limits_{j=1}^{m} \Big[a_{ji} f_{1j}(y_j(t)) + d_{ji} \int\limits_0^\infty k_{ji}(s) f_{2j}(y_j(t-s))ds\Big] + I_i,\ i = 1, 2, ..., n, \\ y_j'(t) = -r_j(y_j(t)) + \sum\limits_{i=1}^{n} \Big[\bar{a}_{ij} g_{1i}(x_i(t)) + \bar{d}_{ij} \int\limits_0^\infty h_{ij}(s) g_{2i}(x_i(t-s))ds\Big] + J_j,\ j = 1, 2, ..., m, \\ x_i(t) = \phi_i(t),\ t \le 0, \\ y_j(t) = \varphi_j(t),\ t \le 0. \end{cases} \tag{65}$$

To design such networks, it is crucial to study the qualitative behavior of their solutions, which practically means that small changes in input signals, initial data or system parameters do not result in significant changes in the state of the system. Besides, unstable neural

networks have no sense in practice. Moreover, the activation functions, which connect the inputs to the outputs of the networks, represent a basic element of artificial neural networks. In general, these function of hidden neurons present a degree of non-linearity which is important in the majority of applications for artificial neural networks. Initially, they were supposed bounded, smooth and monotonic functions [3]. Later, there was a slight relaxation of these conditions to be of Lipschitz type, which has been widely considered in the literature [1]. Due to the significance of non-Lipschitz activation functions in implementations, relaxing the Lipschitz condition is required. Through a more relaxed condition on such functions, in this talk, we derive sufficient conditions to ensure the exponential stability using a nonlinear Halanay inequality as well as some analytical techniques. The effectiveness of the theoretical results is validated by a numerical example.

Keywords: Exponential stability, BAM neural network, distributed delay, nonlinear Halanay inequality.

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Inequalities of the numerical radius for operator matrices and its applications

Soumia Soltani [(1)], Abdekader Frakis [(2)]

[(1)] Department of mathematics, University Mustapha Stambouli of Mascara
E-mail: soumia.soltani@univ-mascara.dz

[(2)] Department of mathematics, University Mustapha Stambouli of Mascara
E-mail: frakis.aek@univ-mascara.dz

Abstract: In this talk, we present some upper and lower bounds of numerical radius for a bounded linear operator matrices defined on a complex Hilbert space $\mathcal{H}$. We also apply this results to estimate the numerical radius of Frobenius Companion matrices for monic polynomial of degree $n \geq 2$ with complex coefficients.

In the present talk, we consider the numerical radius which defined as

$$w(A) = \sup_{\|x\|=1} |\langle Ax, x\rangle|,$$

where A is a bounded linear operator on a complex Hilbert space $\mathcal{H}$ (i.e $A \in \mathbb{B}(\mathcal{H})$) with inner product $\langle \cdot, \cdot \rangle$. The following inequality

$$\frac{1}{2}\|A\| \leq w(A) \leq \|A\|, \quad \text{for every } A \in \mathbb{B}(\mathcal{H}),$$

give to as the equivalent between the numerical radius and the usual operator norm $\|\cdot\|$. Different researchers was provided the both side of this inequality (Lower and upper).

We have $p(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_1 + a_0$ be a monic polynomial of degree $n \geq 2$ with complex coefficent $a_0, a_1, \ldots, a_{n-1}$. Then the Frobenius companion matrix of p is the matrix

$$C(p) = \begin{bmatrix} -a_n & -a_{n-1} & \ldots & -a_2 & -a_1 \\ 1 & 0 & \ldots & \ldots & 0 \\ 0 & 1 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ldots & 0 & 1 & 0 \end{bmatrix}.$$

It is well Known that the eigenvalue of $C(p)$ are exactly the zeros of the polynomial $p(z)$. Since z is any zero of the polynomial $p(z)$, it follows that

$$|z| \leq w(C(p)) \quad as \quad \sigma(C(p)) \subseteq \overline{W(C(p))}. \tag{66}$$

The refinement result was used to estimate the lower bound of $|\lambda|$, which λ is eigenvalue, of Frobenius Companion matrix.

Keywords: Crawford number; Numerical radius; normal operator; operator norm; spectral radius.

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Recent Developments in Ordinary and Partial Differential Equations

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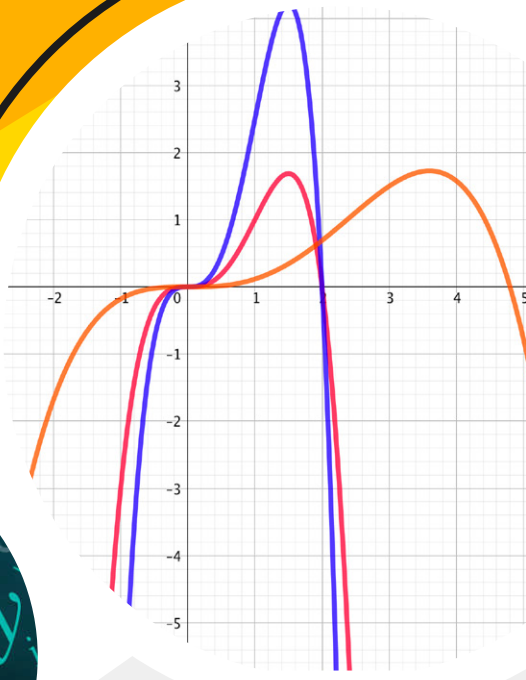

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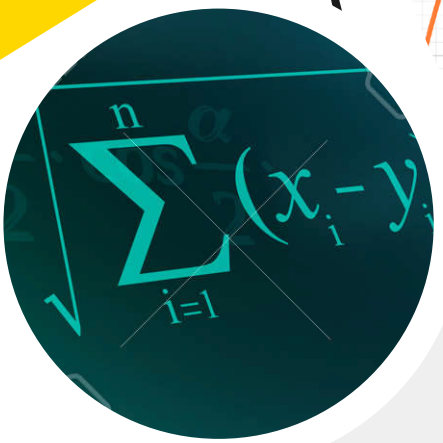

INVITED SPEAKERS

- Irena Lasiecka, Department of Mathematical Sciences, University of Memphis, USA.
- Mokhtar Kirane, University of La Rochelle, La Rochelle, France.
- Tian Xiang, Institute For Mathematical Sciences (IMS) at Renmin University of China.
- Hacène Belbachir, Université des Sciences et de la Technologie Houari Boumediene, Algeria.
- Nasser-eddine Tatar, King Fahd University of Petroleum and Minerals, Saudi Arabia.
- Miyasita Tosiya, Yamato University, Japan.
- Khaled Zennir, Department of Mathematics, College of Science and Arts, Al-Ras, Qassim University, Saudi Arabia.
- Mourad Sini, Radon Institute, Austrian Academy of Sciences, Austria.
- Svetlin Georgiev, University of Sofia, Bulgaria.
- Radu Precup, Babeş-Bolyai University, Cluj-Napoca, Romania.

CONFERENCE TOPICS

- Linear and nonlinear operators in function spaces.
- Differential, integral and operator equations.
- Initial and boundary value problems for ordinary and partial differential equations.
- Numerical methods for ordinary and partial differential equations.
- Mathematical and computer modeling.
- Mathematical physics and modeling in physics.

IMPORTANT DATES

- Registration deadline: 15 April 2022.
- Submission of Abstracts deadline: 15 April 2022.
- Notification for accepted abstracts: 03 May 2022.
- Conference program: 15 May 2022

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